Electromagnetics
Part 2

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Course Overview

1. Plane Waves
2. Polarization
3. Plane Waves in Multilayer Media
4. Excitation Problem and Eigenvalue Problem
5. Wave Physics

1. Plane Waves (1)

Plane waves (homogeneous and sourceless region)

$$\nabla\times\mathbf{E}=-j\omega\mathbf{B}$$
$$\nabla\times\mathbf{H}=j\omega\mathbf{D}$$

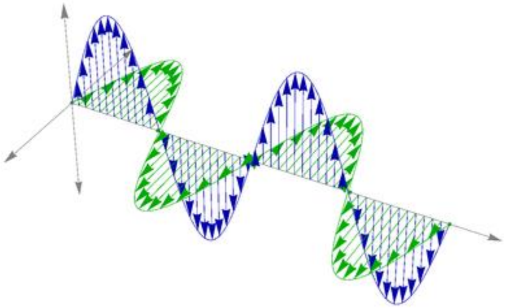
$$\nabla\times\nabla\times\mathbf{E}=-j\omega\nabla\times\mathbf{B}=\omega^2\mu\varepsilon\mathbf{E}$$

$k=\omega\sqrt{\mu\varepsilon}$ wavenumber

$$\nabla\times\nabla\times\mathbf{E}-k^2\mathbf{E}=0$$

$\nabla\cdot\mathbf{E}=0$ sourceless

$$\boxed{\nabla^2\mathbf{E}+k^2\mathbf{E}=0}$$



plane wave is an eigenmode of wave equation (solution of Maxwell equation without excitations)

$$\mathbf{E}=E_0\exp(-jkz)\mathbf{e}_x \xrightarrow{\text{time domain}} \mathbf{E}=E_0\,\mathrm{Re}\left[\exp(j\omega t)\exp(-jkz)\right]\mathbf{e}_x=E_0\cos(\omega t-kz)\mathbf{e}_x$$

E_0 → amplitude

$\exp(-jkz)$ → propagation direction

$\mathbf{e}_x$ → polarization direction

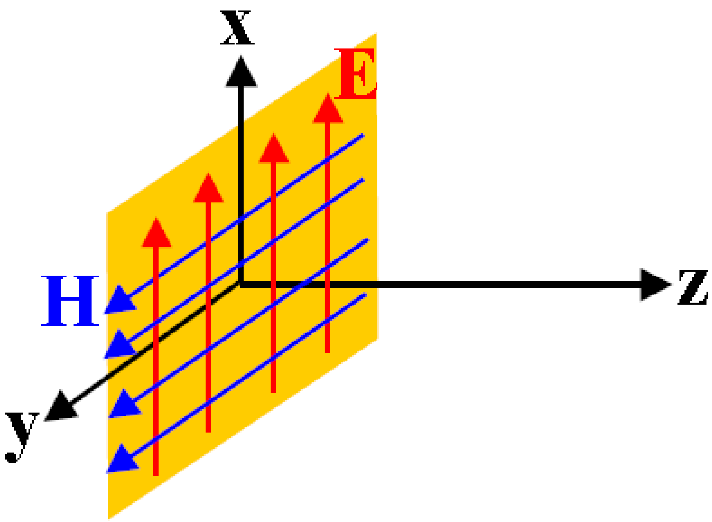
Ref: Xie, Sections 4.1 and 5.1

1. Plane Waves (2)

$$\nabla \times \mathbf{E} = -j\omega \mathbf{B} \longrightarrow \mathbf{H} = \frac{\nabla \times \mathbf{E}}{-j\omega\mu} = \frac{1}{-j\omega\mu}\frac{\partial E_x}{\partial z}\mathbf{e}_y$$

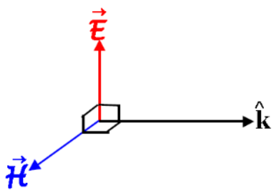
$$= E_0 \frac{k}{\omega\mu}\exp(-jkz)\mathbf{e}_y = \frac{E_0}{\eta}\exp(-jkz)\mathbf{e}_y \longrightarrow \eta = \frac{\omega\mu}{k} = \sqrt{\frac{\mu}{\varepsilon}}$$

<u>wave impedance</u>



$$\omega t - k_0 z = C$$

wavefront is defined by setting the phase equal to a constant



$$v_p = \frac{dz}{dt} = \frac{\omega}{k}$$

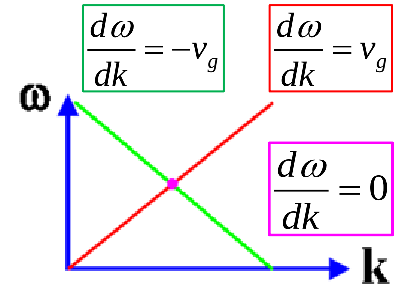
<u>phase velocity</u>

1. Amplitudes of fields on a given constant-phase plane are uniform.
2. In practice, most waves are spherical waves rather than plane waves. But if the observation point is far from the source, it can be approximated as a plane wave.
3. Gauss beam from laser can be approximated as a plane wave.

1. Plane Waves (3)

<u>Propagating waves</u>

$$\mathbf{E} = E_0 \exp(-jkz)\mathbf{e}_x \xrightarrow[\text{domain}]{\text{time}} \mathbf{E} = E_0 \cos(\omega t - kz)\mathbf{e}_x$$

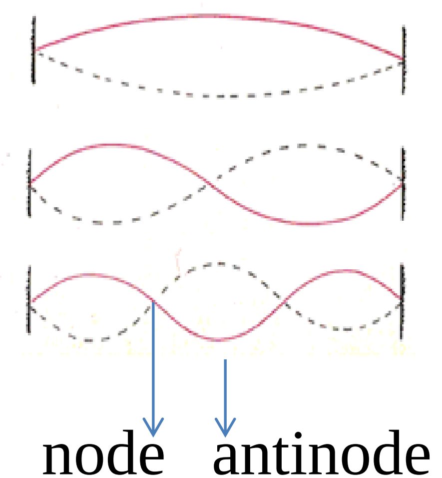


<u>Standing waves</u>

$$\mathbf{E} = 0.5\begin{bmatrix} E_0 \exp(-jkz)\mathbf{e}_x \\ +E_0 \exp(jkz)\mathbf{e}_x \end{bmatrix} \xrightarrow[\text{domain}]{\text{time}} \mathbf{E} = E_0 \cos(\omega t)\cos(kz)\mathbf{e}_x$$

$$= E_0 \cos(kz)\mathbf{e}_x$$

1. Standing waves do not propagate EM energy with zero <u>group velocity</u>.
2. Standing waves relate to eigenmodes of EM resonances.



nodal plane

Mode shape	m	n	$(\omega/\omega_0)^2$	ω/ω_0
+	1	1	1.25	1.12
+ ¦ −	1	2	2.00	1.41
+ ¦ − ¦ +	1	3	3.25	1.80
− / +	2	1	4.25	2.06
− ¦ + / + ¦ −	2	2	5.00	2.24

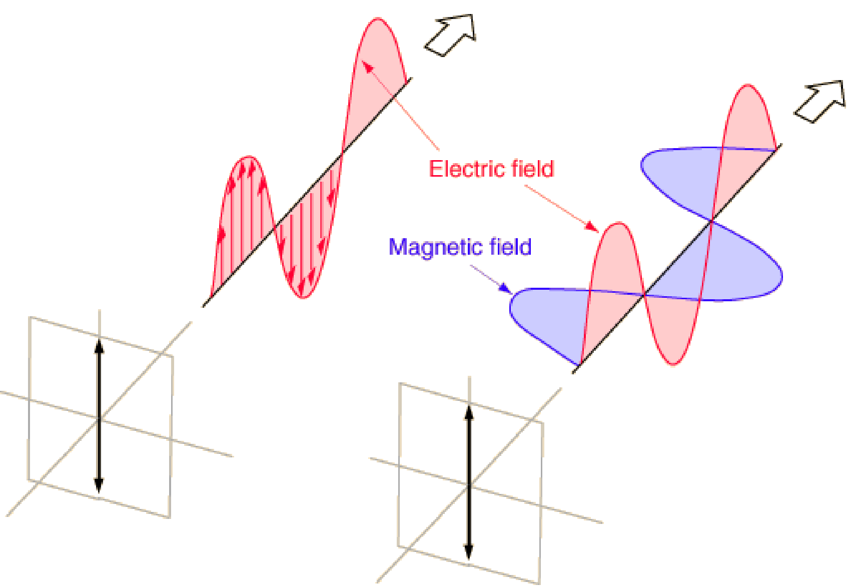
Ref: Xie, Section 5.4 (group velocity); 6.1.1 and 6.1.2 (standing wave)

2. Polarization

The curve traced out by the tip of **E** at a fixed point in space as time t varies

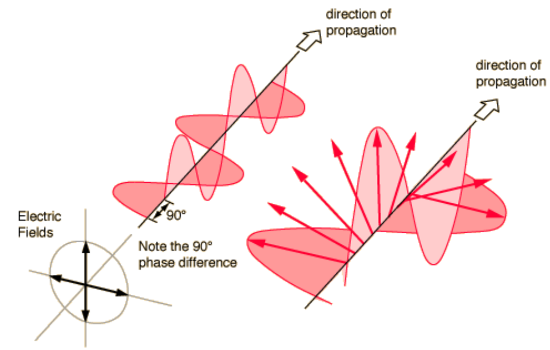
linearly polarized : locus is a straight line
circularly polarized : locus is a circle
elliptically polarized : locus is an ellipse
unpolarized : superposition of linearly polarized waves with random orientations (Sunlight).

linear polarized



right-circularly polarized

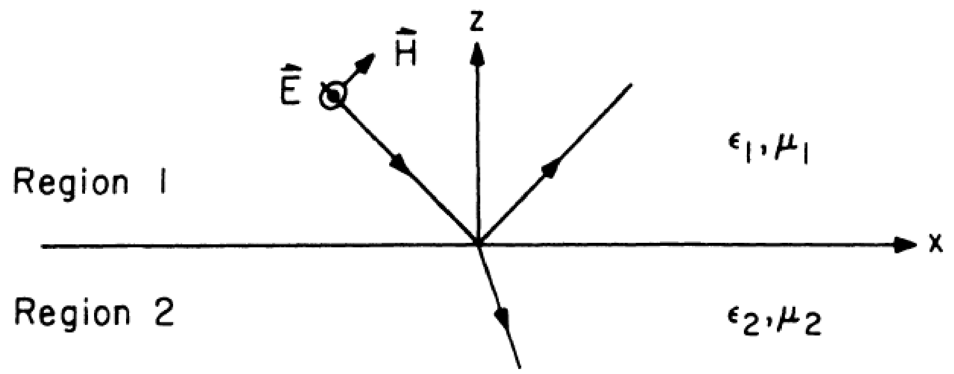
$$\mathbf{E} = \left(\mathbf{e}_x - j\mathbf{e}_y\right) E_0 \exp(-jkz)$$



If looking at the source, electric vector coming toward you is rotating counterclockwise, the wave is right-circularly polarized. If clockwise, it is left-circularly polarized.

3. Plane Waves in Multilayer Media (1)

Reflection and transmission of a plane wave at an interface



TE waves $E_y = e_y(z) e^{-jk_x x}$ $\quad e_{1y}(z) = e_0 \exp(jk_{1z} z) + R^{TE} e_0 \exp(-jk_{1z} z)$

$$E_{1t} = E_{2t}$$
$$H_{1t} = H_{2t}$$

→

$$1 + R^{TE} = T^{TE}$$
$$\frac{k_{1z}}{\mu_1}\left(1 - R^{TE}\right) = \frac{k_{2z}}{\mu_2} T^{TE}$$

→

$$R^{TE} = \frac{\mu_2 k_{1z} - \mu_1 k_{2z}}{\mu_2 k_{1z} + \mu_1 k_{2z}}$$
$$T^{TE} = \frac{2\mu_2 k_{1z}}{\mu_2 k_{1z} + \mu_1 k_{2z}}$$

$$R^{TM} = \frac{\varepsilon_2 k_{1z} - \varepsilon_1 k_{2z}}{\varepsilon_2 k_{1z} + \varepsilon_1 k_{2z}}$$
$$T^{TM} = \frac{2\varepsilon_2 k_{1z}}{\varepsilon_2 k_{1z} + \varepsilon_1 k_{2z}}$$

Ref: Chew, Waves and Fields in Inhomogeneous Media, Chapter 2.

3. Plane Waves in Multilayer Media (2)

1. If $k_1 > k_2$, there exist values of k_x such that $k_1 > k_x > k_2$, implying that k_{2z} is purely imaginary, while k_{1z} is purely real. Hence, the magnitude of R_{TE} or R_{TM} equals 1. In other words, all the energy of the incident wave is reflected. This phenomenon is known as ***total internal reflection***.

2. there exist values of k_x such that R_{TE} or R_{TM} equals zero. Then, the corresponding angle for which the reflection coefficient equals zero is known as the ***Brewster angle.*** Brewster angle effect is more prevalent for TM waves because most materials are nonmagnetic.

3. The poles of the reflection coefficients relate to dispersion relation of the multilayer system. (For eigenvalue problem, we have the nonzero reflection field with a zero incident field/excitation!)

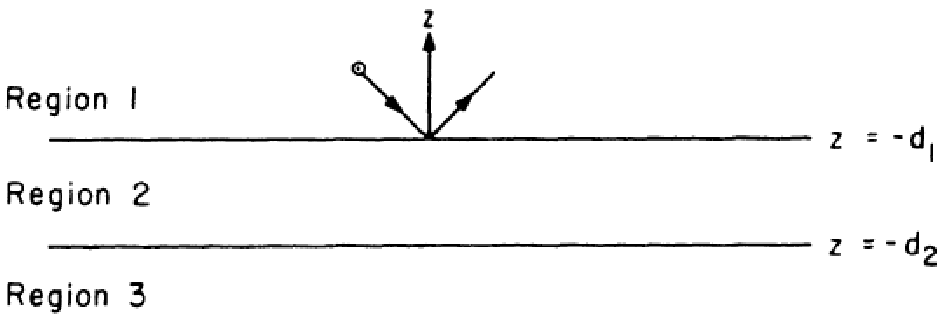
TM wave $\varepsilon_2 k_{1z} + \varepsilon_1 k_{2z} = 0$ $\varepsilon_2 \sqrt{k_1^2 - k_x^2} + \varepsilon_1 \sqrt{k_2^2 - k_x^2} = 0$

$$k_x = k_0 \sqrt{\frac{\varepsilon_1 \varepsilon_2}{\varepsilon_1 + \varepsilon_2}}$$ dispersion of surface plasmon!

 Ref: Kasap, Second Ed. Section 1.5 and 1.6.

3. Plane Waves in Multilayer Media (3)

Reflection from a three-layer medium



wave in region 1

$$e_{1y} = A_1[\exp(jk_{1z}z) + \tilde{R}_{12}\exp(-2jk_{1z}d_1 - jk_{1z}z)]$$

<u>generalized reflection coefficient</u>

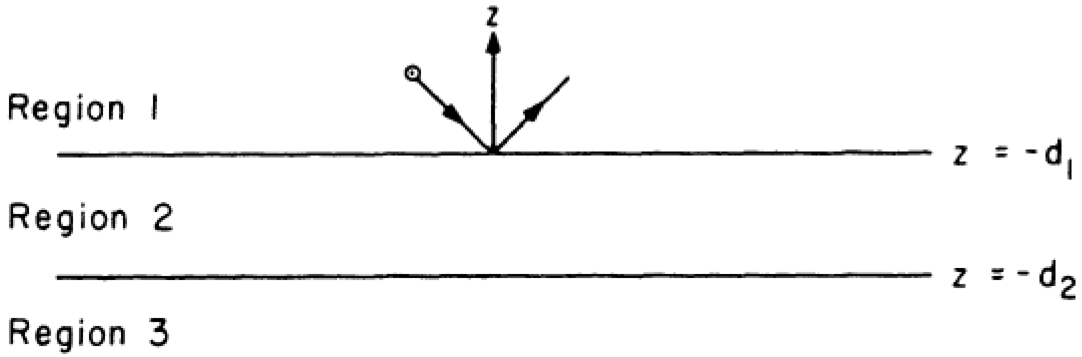
wave in region 2

$$e_{2y} = A_2[\exp(jk_{2z}z) + R_{23}\exp(-2jk_{2z}d_2 - jk_{2z}z)]$$

wave in region 3

$$e_{3y} = A_3\exp(jk_{3z}z)$$

3. Plane Waves in Multilayer Media (4)

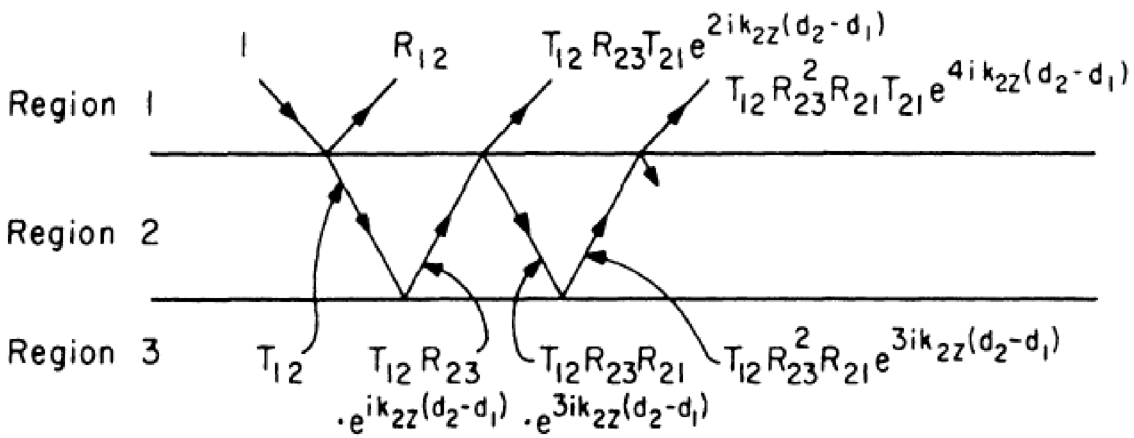


1. Downgoing wave in region 2 is a consequence of the transmission of downgoing wave in region 1 plus a reflection of upgoing wave in region 2 (at interface of $-d_1$)

$$A_2 \exp(-jk_{2z}d_1) = A_1 \exp(-jk_{1z}d_1)T_{12} + R_{21}A_2R_{23}\exp(-2jk_{2z}d_2 + jk_{2z}d_1)$$

2. Upgoing wave in region 1 is caused by the reflection of downgoing wave in region 1 plus a transmission of upgoing wave in region 2 (at interface of –d1)

$$A_1\tilde{R}_{12}\exp(-jk_{1z}d_1) = R_{12}A_1\exp(-jk_{1z}d_1) + T_{21}A_2R_{23}\exp(-2jk_{2z}d_2 + jk_{2z}d_1)$$

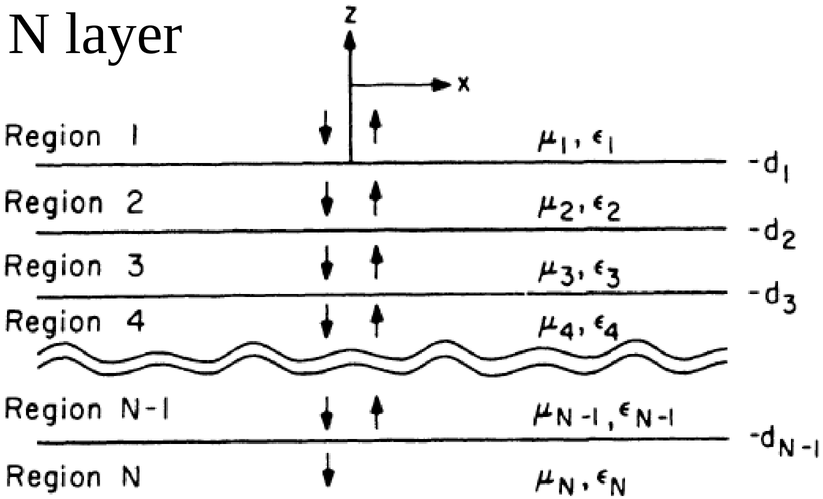


$i = -j$

$$\tilde{R}_{12} = R_{12} + \frac{T_{12} R_{23} T_{21} \exp\left[-2 j k_{2z} (d_2 - d_1)\right]}{1 - R_{21} R_{23} \exp\left[-2 j k_{2z} (d_2 - d_1)\right]}$$

$$\tilde{R}_{12} = R_{12} + T_{12} R_{23} T_{21} \exp\left[-2 j k_{2z} (d_2 - d_1)\right] + T_{12} R_{23}^2 R_{21} T_{21} \exp\left[-4 j k_{2z} (d_2 - d_1)\right] + \ldots$$

3. Plane Waves in Multilayer Media (6)



$$\tilde{R}_{12} = R_{12} + \frac{T_{12} R_{23} T_{21} \exp\left[-2jk_{2z}(d_2 - d_1)\right]}{1 - R_{21} R_{23} \exp\left[-2jk_{2z}(d_2 - d_1)\right]}$$

generalized

$$\tilde{R}_{i,i+1} = R_{i,i+1} + \frac{T_{i,i+1} \tilde{R}_{i+1,i+2} T_{i+1,i} \exp\left[-2jk_{i+1,z}(d_{i+1} - d_i)\right]}{1 - R_{i+1,i} \tilde{R}_{i+1,i+2} \exp\left[-2jk_{i+1,z}(d_{i+1} - d_i)\right]}$$

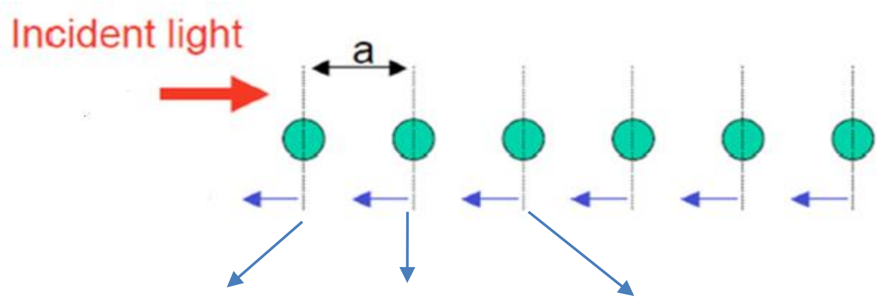
Recursive equation for generalized reflection coefficient

$R_{i,j} = -R_{j,i}$ $\qquad T_{i,j} = 1 + R_{i,j}$ $\qquad$ (See Slide 7)

$$\tilde{R}_{i,i+1} = \frac{R_{i,i+1} + \tilde{R}_{i+1,i+2} \exp\left[-2jk_{i+1,z}(d_{i+1} - d_i)\right]}{1 + R_{i,i+1} \tilde{R}_{i+1,i+2} \exp\left[-2jk_{i+1,z}(d_{i+1} - d_i)\right]}$$

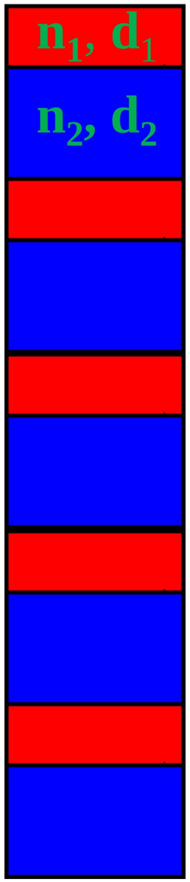
$$\tilde{R}_{N,N+1} = 0$$

Distributed Bragg reflector



$$\tilde{R} = R + (1\text{-}R)Re^{-2jka} + (1\text{-}R)^2 Re^{-4jka} + \cdots$$

$$= \frac{R}{1-(1-R)e^{-2jka}}$$



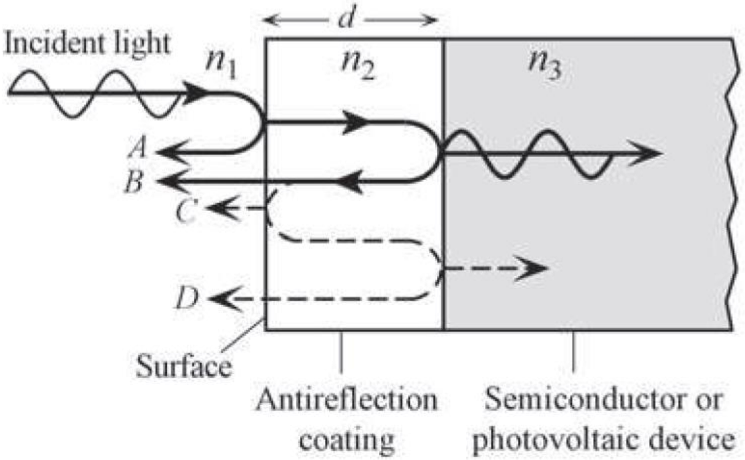
5 periods

when exp(-2*jka*)=1, generalized reflection coefficient will equal to 1 due to constructive inference. Bragg condition is **a=λ/2**. Here we ignore the secondary reflection/transmission. The above can be generalized to 1D multilayer media, i.e.

$$\mathbf{n_1 d_1 + n_2 d_2 = \lambda/2}.$$

 Ref: Kasap, Second Ed. Page 56.

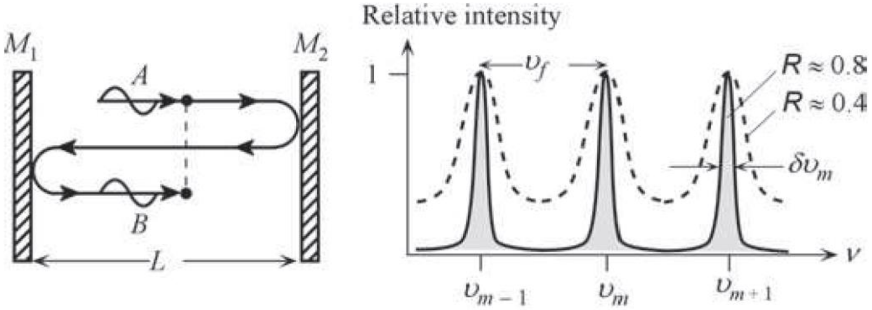
More Interference Cases: Antireflection Coatings and Fabry-Perot mode



$$k_2(2d) = \pi$$

$$d = m\left(\frac{\lambda}{4n_2}\right) \qquad m = 1, 3, 5, \ldots$$

$$R = r^2$$



$$k_0(2L) = 2\pi$$

$$m\left(\frac{\lambda}{2}\right) = L; \quad m = 1, 2, 3, \ldots$$

$$\begin{aligned} E_{\text{cavity}} &= A + B + \ldots \\ &= A + Ar^2\exp(-j2kL) + Ar^4\exp(-j4kL) + Ar^6\exp(-j6kL) + \ldots \end{aligned}$$

$$E_{\text{cavity}} = \frac{A}{1 - r^2\exp(-j2kL)}$$

Ref: Kasap, Second Ed. Section 1.7, 1.10, and 1.11.

4. Excitation Problem and Eigenvalue Problem (1)

Eigenvalue problem

$$\nabla \times \nabla \times \mathbf{F}_m(\mathbf{r}) - k_m^2 \mathbf{F}_m(\mathbf{r}) = 0$$

Excitation problem

$$\nabla \times \nabla \times \mathbf{E}(\mathbf{r}) - k_0^2 \mathbf{E}(\mathbf{r}) = i\omega\mu \mathbf{J}(\mathbf{r})$$

How to connect excitation problem with eigenvalue problem?

eigenmode expansion

$$\mathbf{E}(\mathbf{r}) = \sum_m a_m \mathbf{F}_m(\mathbf{r})$$

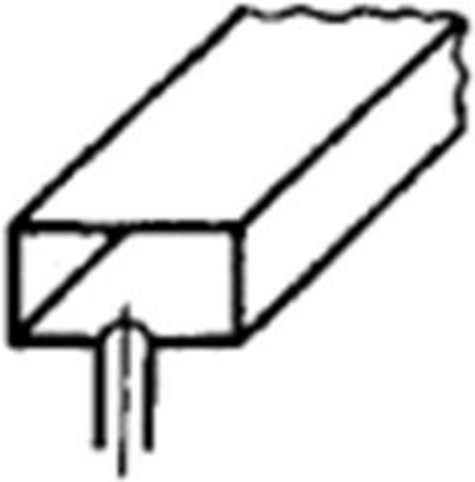
mode orthogonality

$$\int_V d\mathbf{r} \mathbf{F}_{m'}^*(\mathbf{r}) \mathbf{F}_m(\mathbf{r}) = \delta_{m'm}$$

$$\sum_m a_m (k_m^2 - k_0^2) \mathbf{F}_m(\mathbf{r}) = i\omega\mu \mathbf{J}(\mathbf{r})$$

$$a_m = i\omega\mu \frac{\langle \mathbf{F}_m^*, \mathbf{J} \rangle}{k_m^2 - k_0^2}$$

4. Excitation Problem and Eigenvalue Problem (2)

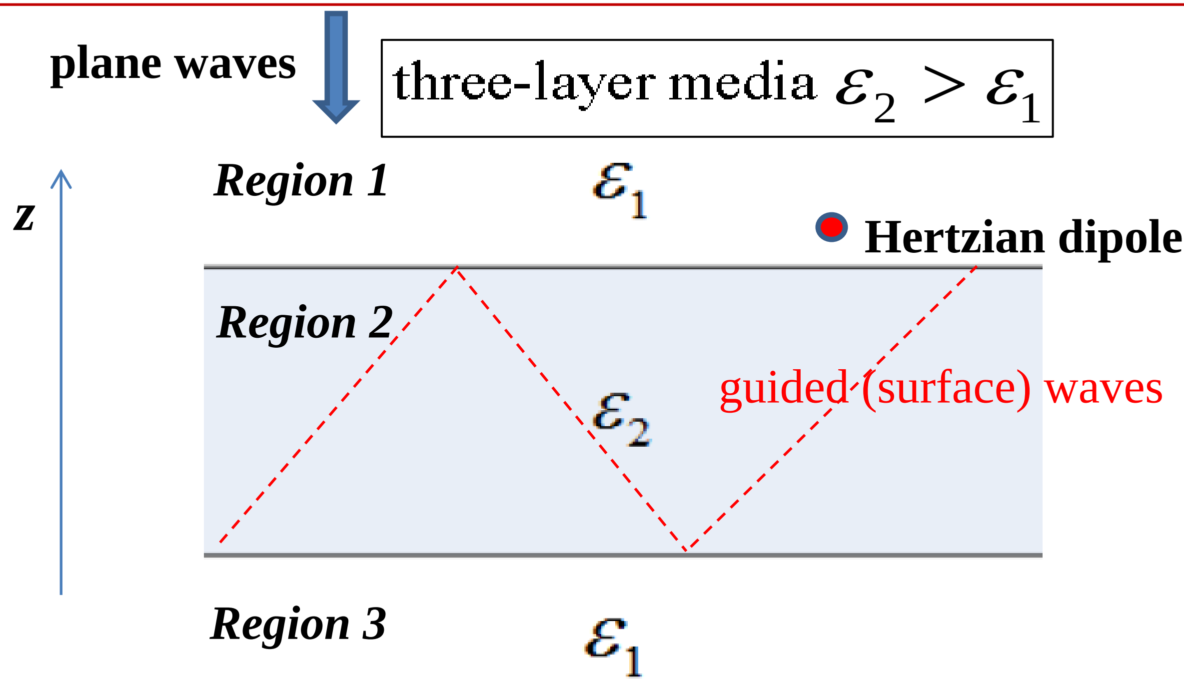


To excite TE_{10} mode of rectangular waveguide
a. Probe is vertically oriented
b. Probe is located at the center of bottom side
Please give me a reason.

1. The above expression tells us if we want a certain mode to be strongly excited, we need the inner product of the mode and current to be large. Hence, the current on the probe should be located at where the field of the mode is strong. If the probe is a short wire, it can be approximated by an electric dipole and its polarization should be align to that of eigenmode.

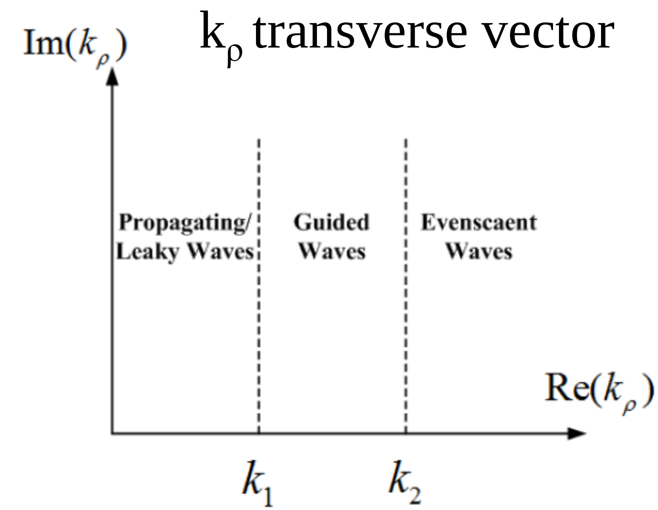
2. If the operating frequency of the source is closed to the resonant frequency of the mode, that mode will be strongly excited. This is the phenomenon of resonance coupling. (waveguide here is not a resonator)

5. Wave Physics (1)



$k_\rho^2 + k_{z1}^2 = k_1^2$ Region 1 and 3

$k_\rho^2 + k_{z2}^2 = k_2^2$ Region 2



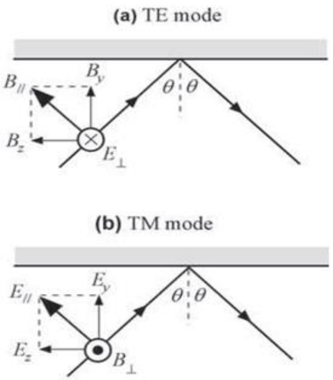
1. Why a plane wave from region 1 cannot excite guided wave propagating in region 2?

Wavenumber of plane wave is smaller than propagation constant of the guided wave (<u>phase mismatch</u>).

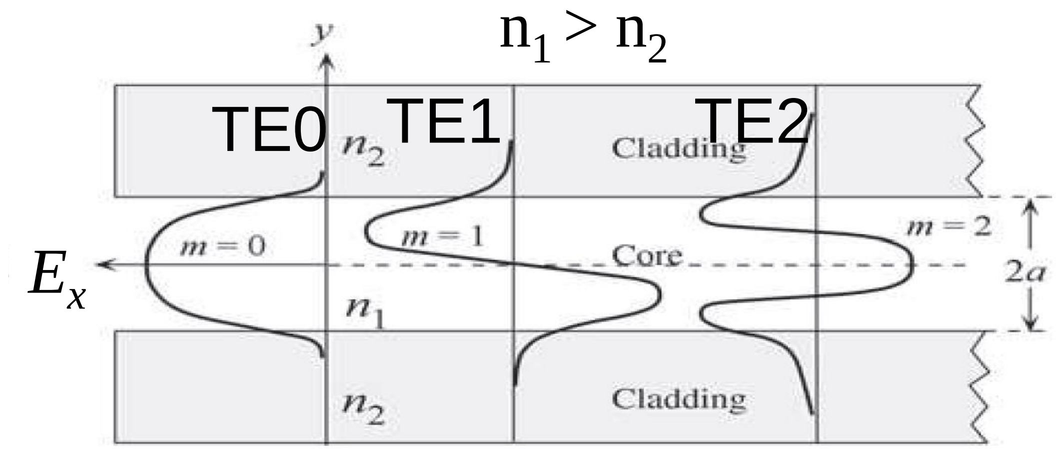
2. Why a Hertzian dipole from region 1 could excite guided waves?

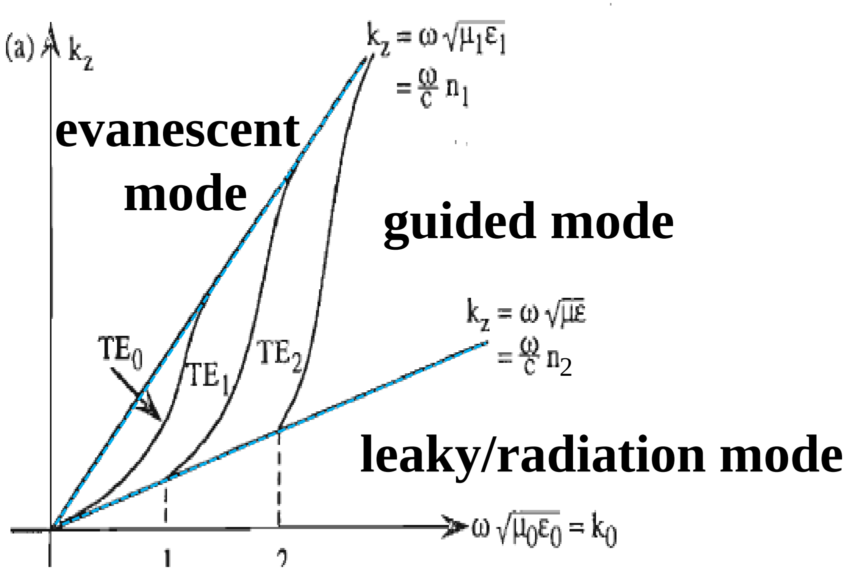
Hertzian dipole with rich evanescent wave components could excite the guided wave if it is near from the interface.

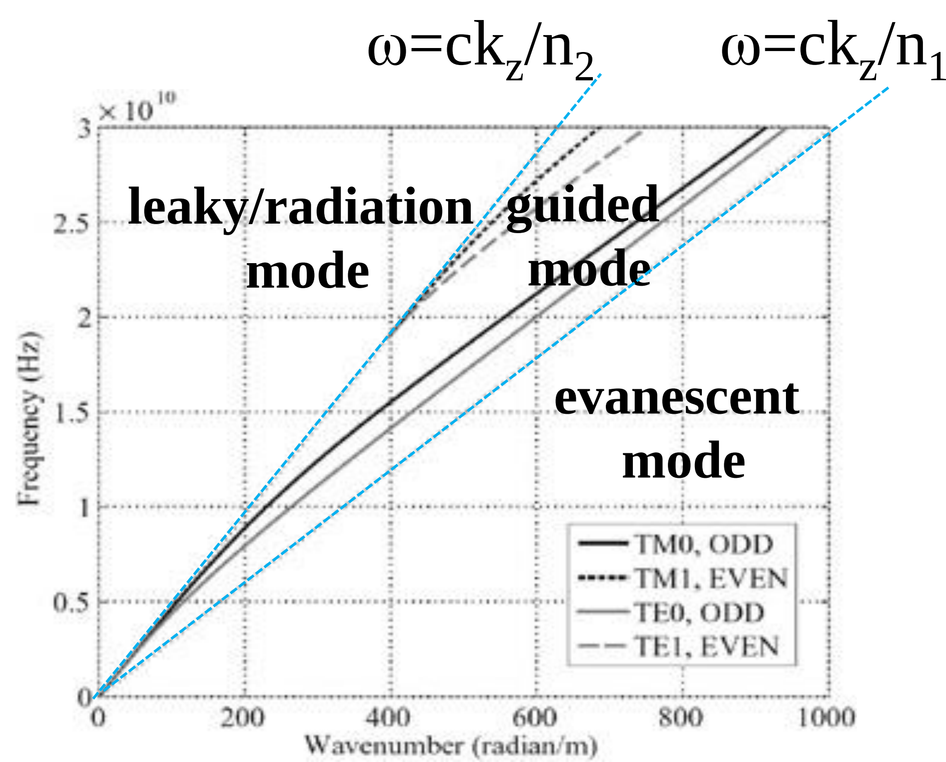
Guided Mode



z



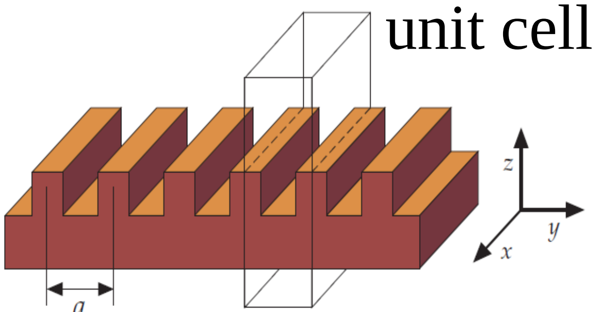




 Ref: Kasap, Second Ed. Section 2.1.

Periodic Grating: We still have continuous translational symmetry in the x direction, but now we have discrete translational symmetry in the y direction

Floquet/Bloch mode

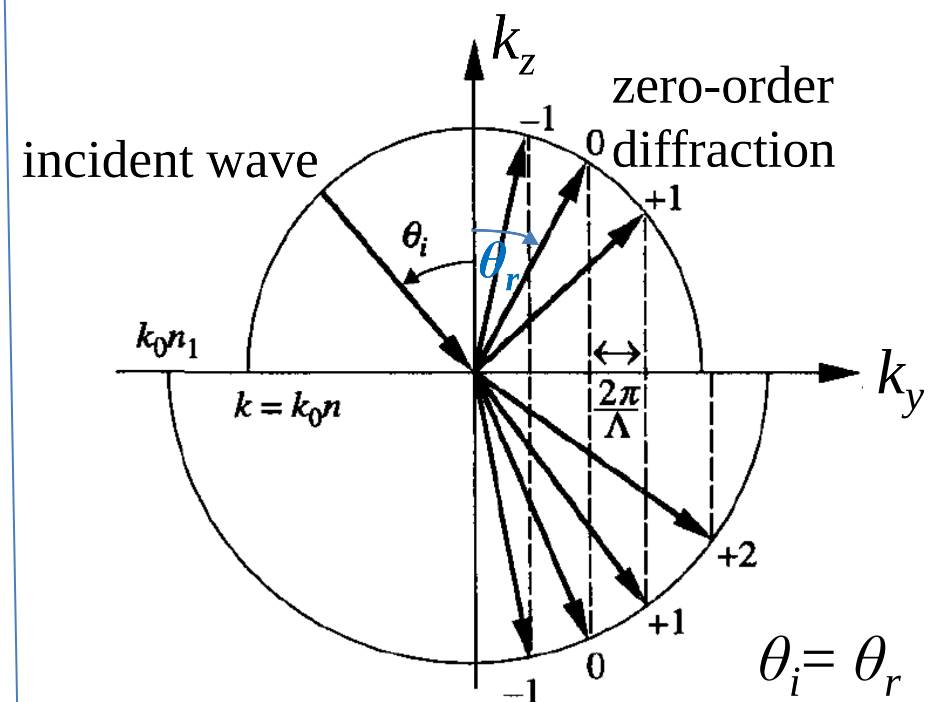


lattice constant

$$T_R \exp\left[i(k_y + 2\pi m/a)y\right]$$
$$= \exp\left[i(k_y + 2\pi m/a)(y - la)\right]$$
$$= \exp\left[i(k_y + 2\pi m/a)y\right]\exp\left[-ik_y la\right]$$

all of the modes with wave vectors of the form $k_y + m(2\pi/a)$, where m is an integer, form a degenerate set; they all have the same eigenvalues of $\boldsymbol{T}_{R.}$

phase matching condition



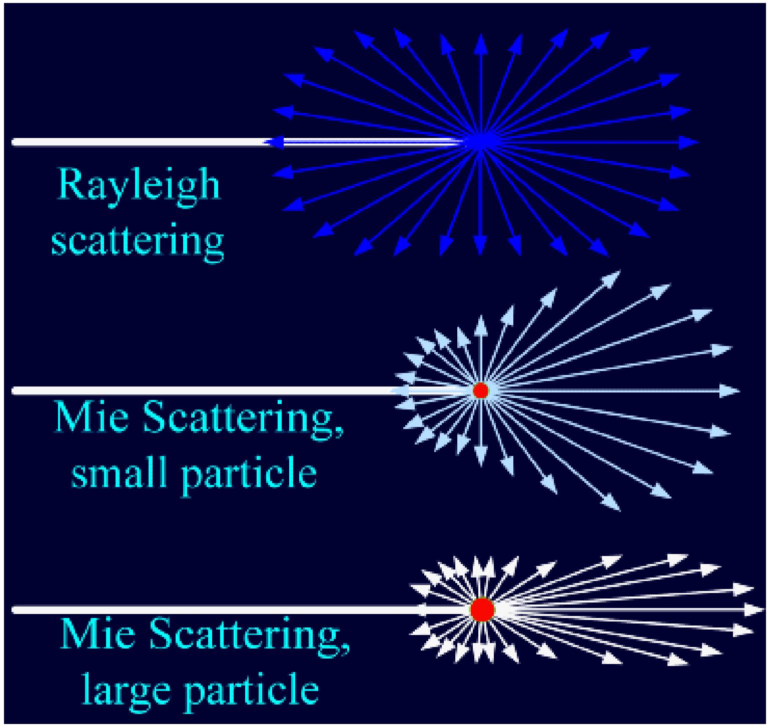
Ref: Kasap, Second Ed., Page. 80-84

Wei SHA

Particle scattering (Optional)

blue sky

sunset



particle size from small to large

Rayleigh scattering ~ $1/\lambda^4$

Mie scattering ~ rely little on wavelength