Future of Electromagnetic Education

Wei E.I. Sha[1] (沙威) and Zhixiang Huang[2] (黄志祥)

1. College of Information Science & Electronic Engineering
***Zhejiang University*, Hangzhou 310027, China**

2. Key Laboratory of Intelligent Computing and Signal Processing, Ministry of Education, *Anhui University*, Hefei 230039, China

Email: weisha@zju.edu.cn (Sha) zxhuang@ahu.edu.cn (Huang)

Oral Session: Room 3311 | May 22 | 10:45 - 11:15 AM

Contents

Teaching — Optical Theorem

Microwave | *Optical (nanoelectromagnetics)*

radar cross section → **absorption, scattering, and extinction cross section**

scattering

$$\sigma_s = \frac{\frac{1}{2}\int_s \mathrm{Re}\left[\mathbf{E}^s \times \mathrm{conj}\left(\mathbf{H}^s\right)\right]\cdot d\mathbf{S}}{|\mathbf{S}_i|}$$

absorption

$$\sigma_a = -\frac{\frac{1}{2}\int_s \mathrm{Re}\left[\mathbf{E} \times \mathrm{conj}\left(\mathbf{H}\right)\right]\cdot d\mathbf{S}}{|\mathbf{S}_i|}$$

extinction

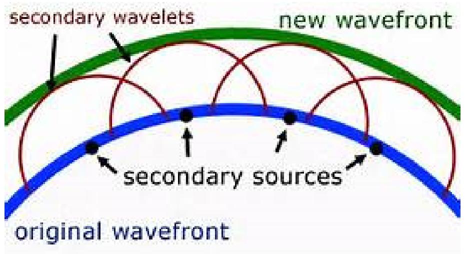
$$\sigma_e = \sigma_s + \sigma_a = -\frac{\frac{1}{2}\int_s \mathrm{Re}\left[\mathbf{E}^i \times \mathrm{conj}\left(\mathbf{H}^s\right) + \mathbf{E}^s \times \mathrm{conj}\left(\mathbf{H}^i\right)\right]\cdot d\mathbf{S}}{|\mathbf{S}_i|}$$

optical theorem

$$\sigma_e = \frac{4\pi}{k_0}\mathrm{Im}[\mathbf{e}^i \cdot \overline{\mathbf{F}} \cdot \mathbf{e}^i]$$

$$\mathbf{E}^s = \frac{\exp(-jk_0 r)}{r}\overline{\mathbf{F}} \cdot \mathbf{E}^i, \quad r \to \infty$$

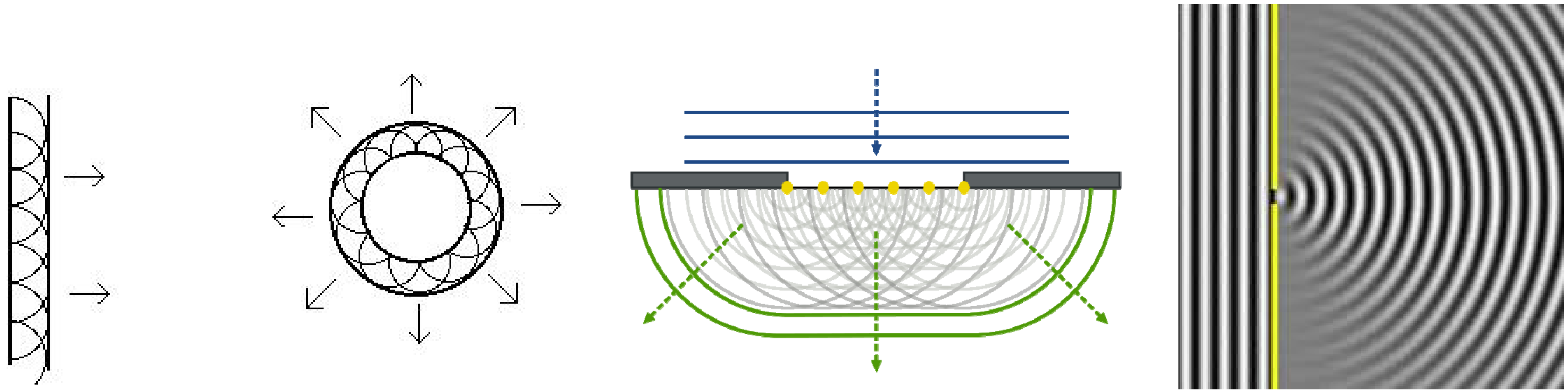
ECS only depends on forward scattering because of orthogonal properties of plane waves!



$$\mathbf{E}(\mathbf{r}) = \int_S \mathbf{M}(\mathbf{r}') \cdot \nabla' \times \bar{\mathbf{G}}(\mathbf{r},\mathbf{r}') dS' - j\omega\mu \int_S \mathbf{J}(\mathbf{r}') \cdot \bar{\mathbf{G}}(\mathbf{r},\mathbf{r}') dS'$$

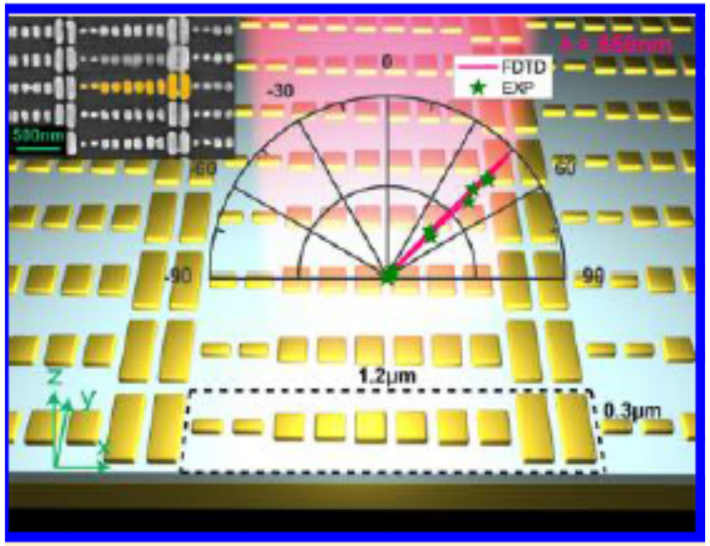
Every point on a wave-front is considered a source of secondary spherical waves which spread out in the forward direction at the speed of light.

<u>Old teaching case</u>

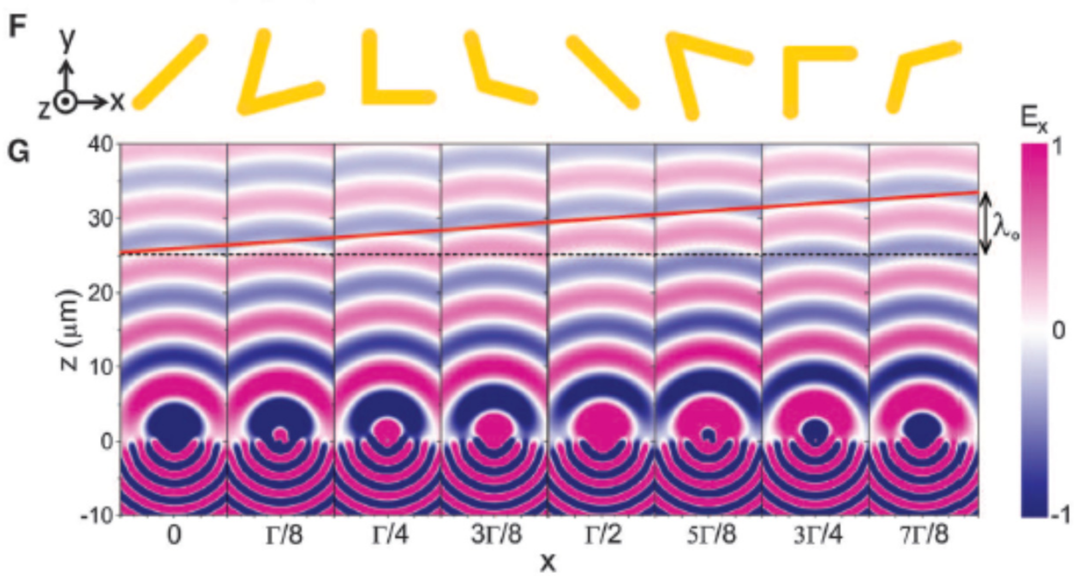


Teaching — Huygens Principle (2)

New teaching case



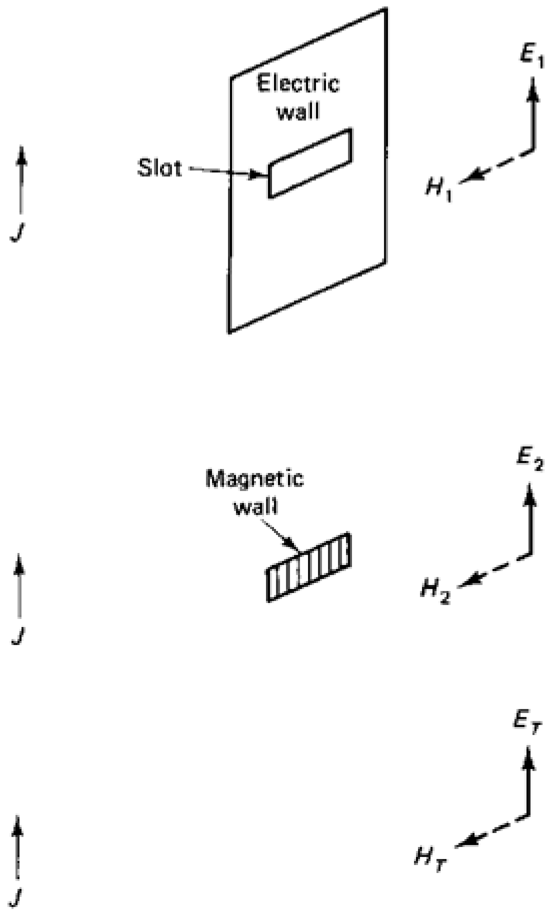
Nano Letters 12, 6223−6229, 2012



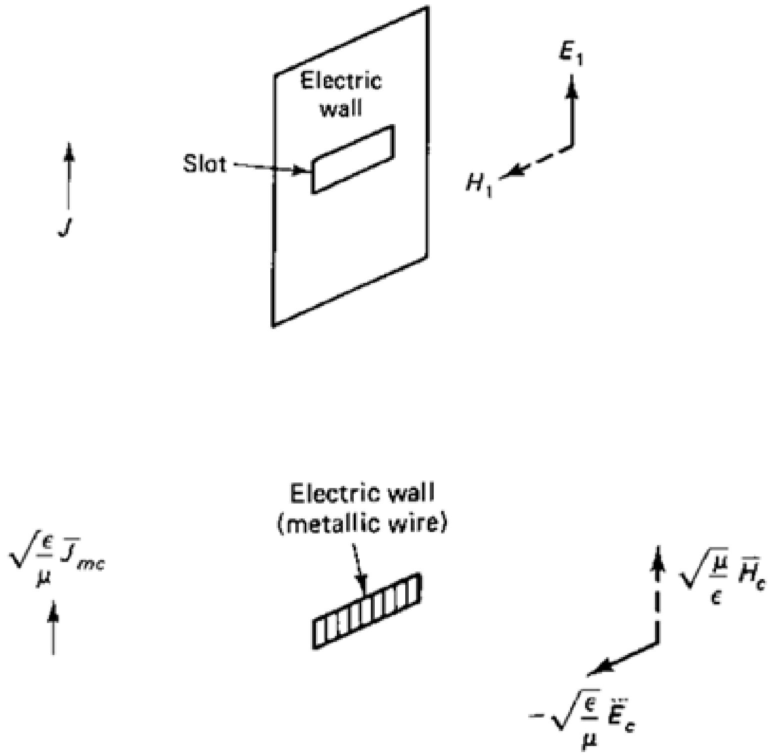
Science 334, 333-337, 2011

From Huygens principle, wave-front of a plane wave can be changed if the phase distribution of corresponding secondary source is modified. Sub-scatters offer the abrupt phase change.

Teaching — Babinet Principle (1)



duality



$$\mathbf{E}_1 + \mathbf{E}_2 = \mathbf{E}_T$$
$$\mathbf{H}_1 + \mathbf{H}_2 = \mathbf{H}_T$$

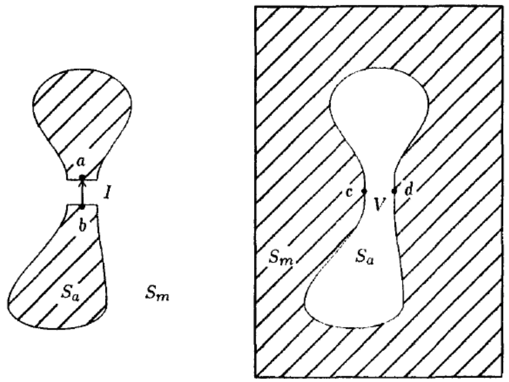
$$\mathbf{E}_1 + \eta \mathbf{H}_c = \mathbf{E}_T$$
$$\mathbf{H}_1 - \frac{1}{\eta}\mathbf{E}_c = \mathbf{H}_T$$

Teaching — Babinet Principle (2)

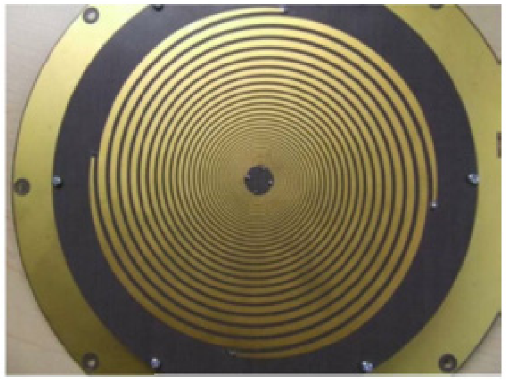
<u>Old teaching case</u>

Input impedances of metal antenna and complementary aperture antenna satisfy

$$Z_m Z_a = \frac{\eta^2}{4}$$

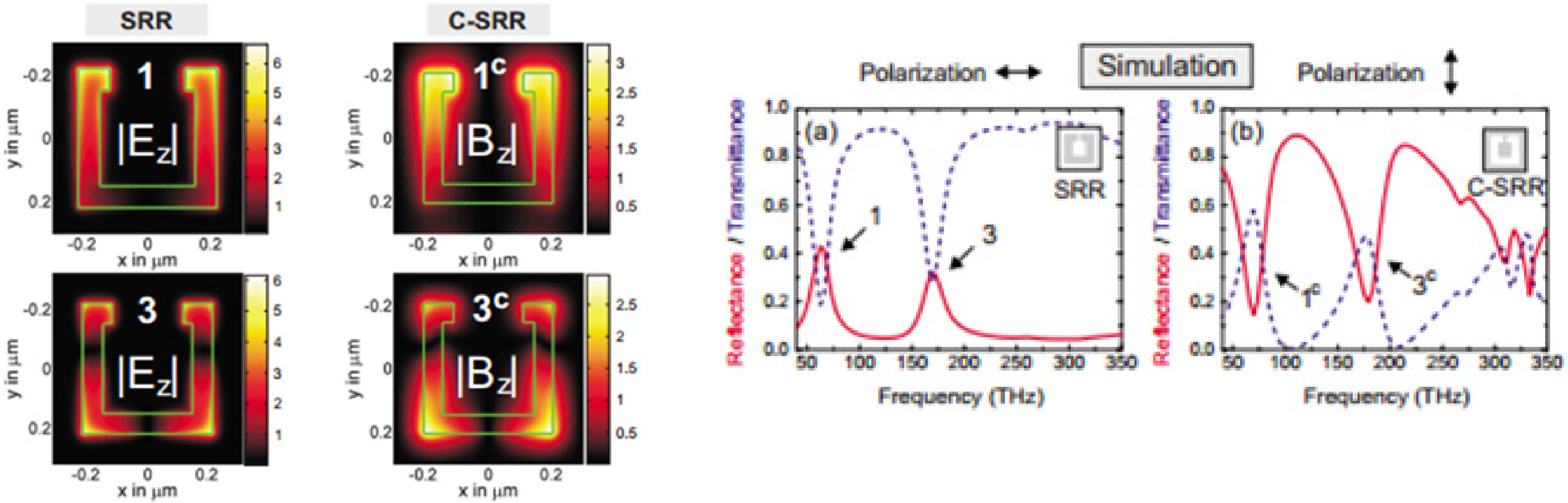


For self-complementary antenna, its input impedance is frequency of independent and thus the self-complementary antenna is a kind of broadband antennas.



Planar Spiral

New teaching case

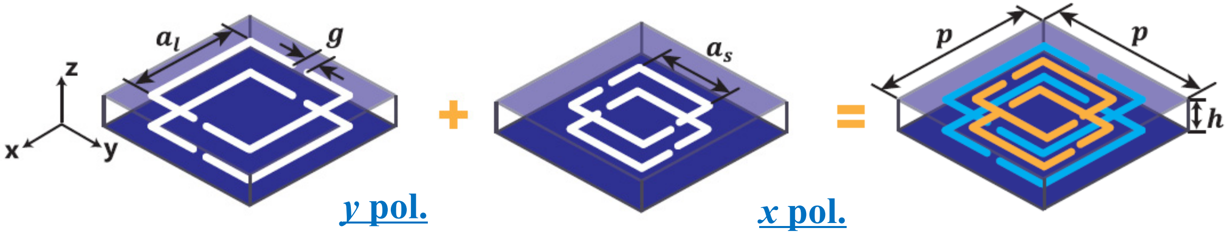


The reflection peak of the split-ring resonator (SRR) is corresponding to the transmission peak of the complementary split-ring resonator (C-SRR).

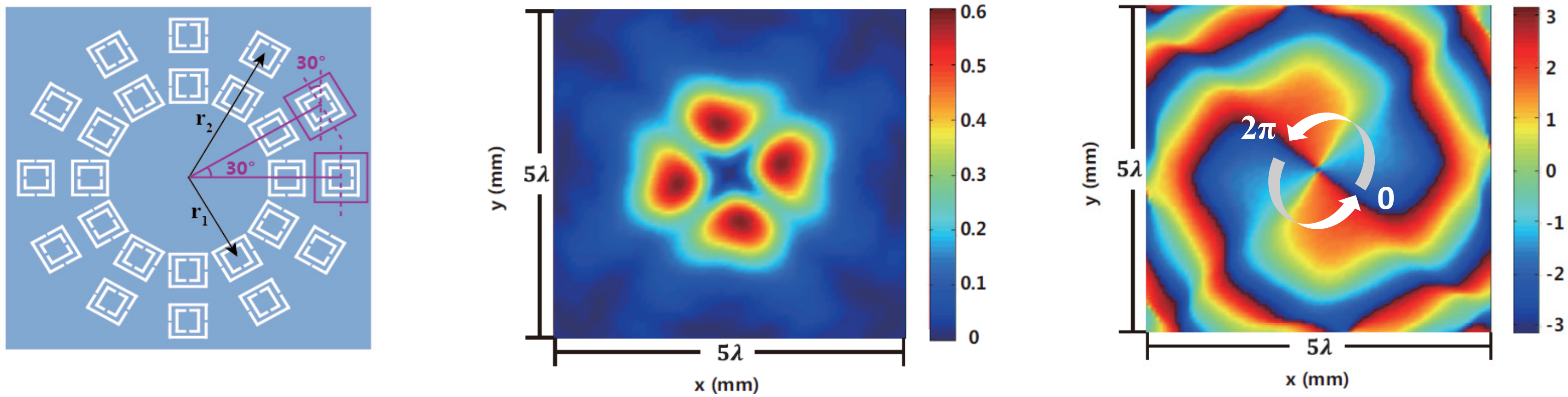
Physical Review B 76, 033407, 2007

Emerging application (Menglin L.N. Chen, Li Jun Jiang, Wei E.I. Sha)

Bi-layer C-SRRs to generate orbital angular momentum of EM waves with a high transmission.



81% conversion efficiency



IEEE Transactions on Antennas and Propagation, 65, 396-400, 2017.

Teaching — Momentum conservation (1)

In engineering EM courses, energy conservation law has been extensively taught as a form of Poynting's theorem. However, momentum conservation law expressed as Maxwell stress tensor form was ignored. The momentum conservation law becomes more and more important for nanoelectromagnetics.

$$\frac{\partial \mathbf{G}_f}{\partial t} + \frac{\partial \mathbf{G}_m}{\partial t} = \int_S \bar{\mathbf{T}} \cdot d\mathbf{S}$$

field momentum

$$\mathbf{G}_f = \frac{1}{c^2} \int_V \mathbf{E} \times \mathbf{H} dV$$

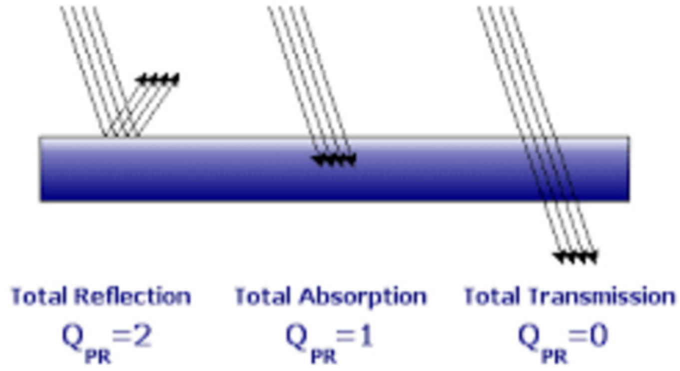
mechanical momentum

$$\frac{\partial \mathbf{G}_m}{\partial t} = \mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

Maxwell's stress tensor

$$\bar{\mathbf{T}} = \varepsilon_0 \mathbf{E}\mathbf{E} - \frac{1}{2}\varepsilon_0 |\mathbf{E}|^2 \bar{\mathbf{I}} + \mu_0 \mathbf{H}\mathbf{H} - \frac{1}{2}\mu_0 |\mathbf{H}|^2 \bar{\mathbf{I}}$$

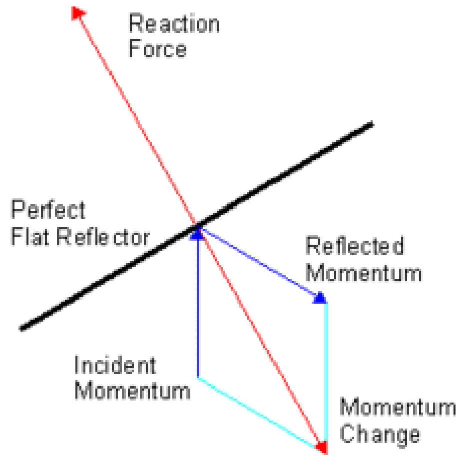
A new look at field momentum



de Broglie hypothesis (wave-particle duality)

$$p = \hbar k = \hbar \frac{\omega}{c} = \frac{E}{c}$$

field momentum

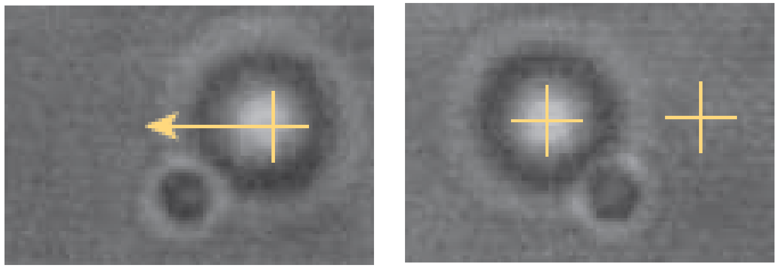


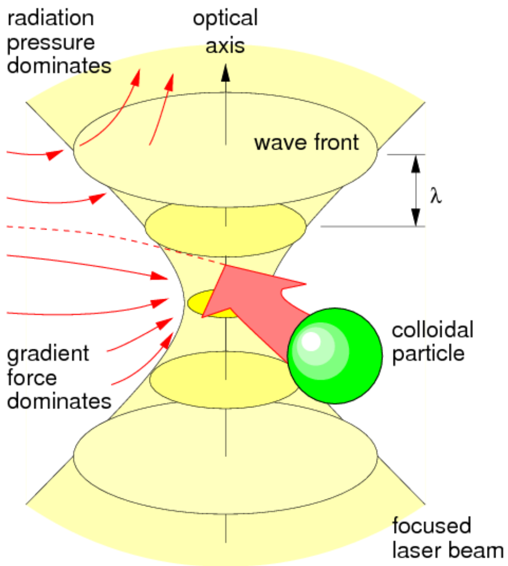
$$\mathbf{G}_f = \mathbf{P}^{em} = \frac{\int (\mathbf{E}\times\mathbf{H})\cdot ds \cdot dt}{c} = \frac{\int (\mathbf{E}\times\mathbf{H})\cdot ds \cdot dl}{c^2} = \frac{\int (\mathbf{E}\times\mathbf{H}) dV}{c^2}$$

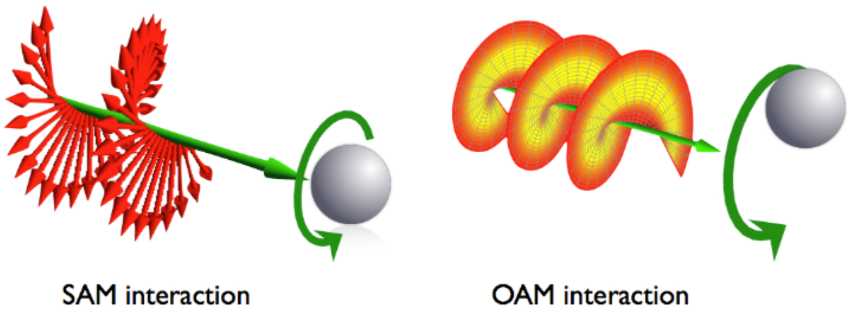
$$\mathbf{G}_f = 1/c^2 \cdot \int (\mathbf{E}\times\mathbf{H}) dV$$

New teaching case

optical tweezer

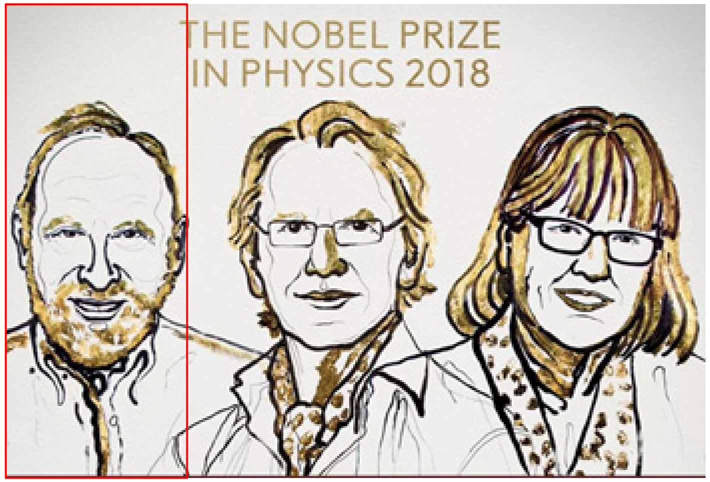






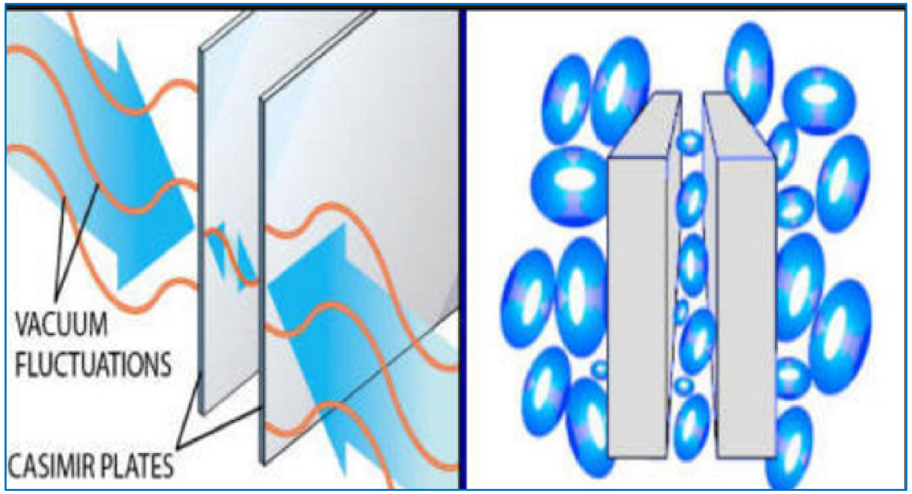
spin and orbital angular momenta

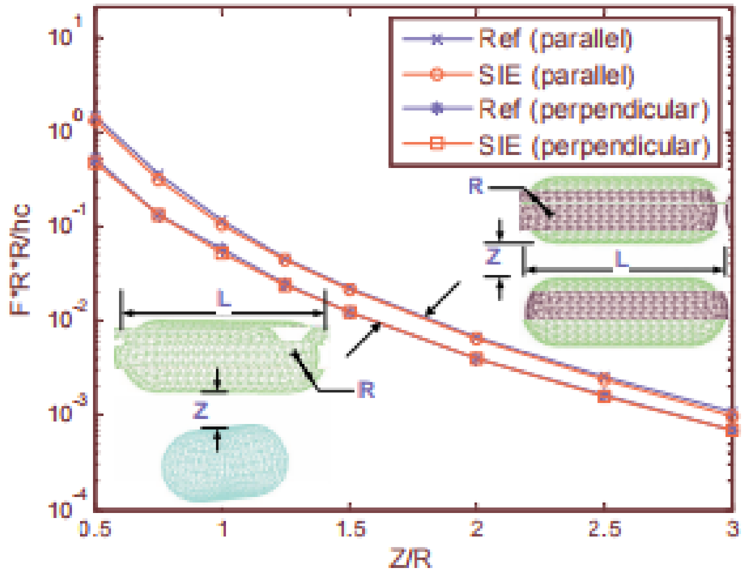
Prof. Arthur Ashkin



<u>Emerging application</u> (Jie Xiong and Weng Cho Chew)

Casimir force calculation by Maxwell stress tensor





$$\mathbf{F} = \oint_S \langle \overline{\mathbf{T}}(\mathbf{r}') \rangle \cdot d\mathbf{s}'$$

$$\langle 0|\hat{E}_i(\mathbf{r},t)\hat{E}_j(\mathbf{r}',t)|0\rangle = \frac{\hbar}{\pi}\mathrm{Im}\int_0^\infty \omega^2 G_{ij}(\mathbf{r},\mathbf{r}',\omega)d\omega$$

$$\langle 0|\hat{B}_i(\mathbf{r},t)\hat{B}_j(\mathbf{r}',t)|0\rangle = \frac{\hbar}{\pi}\mathrm{Im}\int_0^\infty (\nabla\times)_{il}(\nabla'\times)_{jm} G_{lm}(\mathbf{r},\mathbf{r}',\omega)d\omega$$

split of field momentum

$$\mathbf{P}^{em} = 1/c^2 \cdot \int (\mathbf{E}\times\mathbf{H}) dV = \varepsilon_0 \int (\mathbf{E}\times\mathbf{B}) dV$$

longitudinal part

$$\varepsilon_0 \int (\mathbf{E}_{\parallel}\times\mathbf{B}) dV = \int dV \rho(\mathbf{r},t)\mathbf{A}(\mathbf{r},t) = \sum_n q_n(\mathbf{r}_n)\mathbf{A}(\mathbf{r}_n,t)$$

connect to field momentum in quantum mechanics

transverse part

$$\int_V \frac{\varepsilon_0}{2}\mathrm{Re}(\mathbf{E}_{\perp}^{*}\times\mathbf{B}_{\perp}) dV = \int_V \underbrace{\frac{\varepsilon_0}{4\omega}\mathrm{Im}\left[\nabla\times(\mathbf{E}_{\perp}^{*}\times\mathbf{E}_{\perp})\right]}_{\mathbf{p}^s} dV + \int_V \underbrace{\frac{\varepsilon_0}{2\omega}\mathrm{Im}\left[\mathbf{E}_{\perp}^{*}\cdot\nabla\mathbf{E}_{\perp}\right]}_{\mathbf{p}^o} dV$$

spin momentum density orbital momentum density

$$\mathbf{l}^s = \mathbf{r}\times\mathbf{p}^s = \frac{\varepsilon_0}{2\omega}\mathrm{Im}\left[\mathbf{E}_{\perp}^{*}\times\mathbf{E}_{\perp}\right]$$

spin angular momentum density

$$\mathbf{l}^o = \mathbf{r}\times\mathbf{p}^o = \frac{\varepsilon_0}{2\omega}\mathrm{Im}\left[\mathbf{E}_{\perp}^{*}\cdot(\mathbf{r}\times\nabla)\mathbf{E}_{\perp}\right]$$

orbital angular momentum density

Teaching — Orbital Angular Momentum (2)

OAM of photon

$$\mathbf{L}^{o} = \mathrm{Re}\langle \boldsymbol{\psi} | \hat{\mathbf{L}} | \boldsymbol{\psi} \rangle, \quad \boldsymbol{\psi} = \sqrt{\frac{\varepsilon_0}{2\omega\hbar}}\mathbf{E} \rightarrow \hat{\mathbf{E}}$$

Related to Quantum Optics

$$\hat{\mathbf{L}} = \mathbf{r} \times \hat{\mathbf{p}}, \quad \hat{\mathbf{p}} = -i\hbar\nabla$$

OAM of electron

$$\mathbf{L}^{o} = \langle \boldsymbol{\psi} | \hat{\mathbf{L}} | \boldsymbol{\psi} \rangle$$

$$\hat{\mathbf{L}} = \mathbf{r} \times \hat{\mathbf{p}}, \quad \hat{\mathbf{p}} = -i\hbar\nabla$$

SAM of photon

$$\mathbf{L}^{s} = \mathrm{Re}\langle \boldsymbol{\psi} | \hat{\mathbf{S}} | \boldsymbol{\psi} \rangle, \quad \boldsymbol{\psi} = \sqrt{\frac{\varepsilon_0}{2\omega\hbar}}\mathbf{E} \rightarrow \hat{\mathbf{E}}$$

$$\hat{\mathbf{S}}_x = -i\hbar\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}, \quad \hat{\mathbf{S}}_y = -i\hbar\begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix},$$

$$\hat{\mathbf{S}}_z = -i\hbar\begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \left[\hat{\mathbf{S}}_x, \hat{\mathbf{S}}_y\right] = i\hbar\hat{\mathbf{S}}_z$$

SAM of electron

$$\mathbf{L}^{s} = \langle \boldsymbol{\psi} | \hat{\mathbf{S}} | \boldsymbol{\psi} \rangle$$

$$\hat{\mathbf{S}}_x = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \hat{\mathbf{S}}_y = \frac{\hbar}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix},$$

$$\hat{\mathbf{S}}_z = \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \left[\hat{\mathbf{S}}_x, \hat{\mathbf{S}}_y\right] = i\hbar\hat{\mathbf{S}}_z$$

Teaching — Orbital Angular Momentum (3)

Orbital angular momentum in k space

$$\mathbf{L}^o = \int_V \mathrm{Re}\left[\hat{\mathbf{E}}^* \cdot \left(\mathbf{r}\times(-i\hbar\nabla)\right)\hat{\mathbf{E}}\right]dV = \int_{V_k} \mathrm{Re}\left[\hat{\mathbf{E}}^*(\mathbf{k})\cdot\left((-\hbar\mathbf{k})\times i\nabla_k\right)\hat{\mathbf{E}}(\mathbf{k})\right]d\mathbf{k}$$

Berry connection and Berry curvature

$$\mathbf{A}_B(\mathbf{k}) = \left\langle \hat{\mathbf{E}}(\mathbf{k}) \mid i\nabla_k \mid \hat{\mathbf{E}}(\mathbf{k})\right\rangle \qquad \mathbf{F}_B(\mathbf{k}) = \nabla_k \times \left\langle \hat{\mathbf{E}}(\mathbf{k}) \mid i\nabla_k \mid \hat{\mathbf{E}}(\mathbf{k})\right\rangle$$

Berry phase and Chern number

$$\theta_B = \oint_{C_k} \mathbf{A}_B(\mathbf{k})\cdot d\mathbf{l} = \iint_{S_k} \mathbf{F}_B(\mathbf{k})\cdot d\mathbf{k} \qquad C_B = \frac{1}{2\pi}\iint_{S_k} \mathbf{F}_B(\mathbf{k})\cdot d\mathbf{k}$$

Connect to topological charge

$C_B = -(l+\sigma)\dfrac{\Omega}{2\pi}$, Ω is the solid angle of ray contour

$l = \pm 1, \pm 2, \ldots$ topological charge of orbital angular momentum

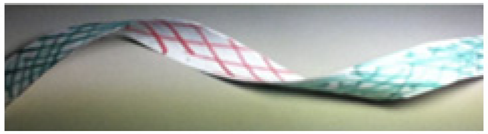
$\sigma = \pm 1$ topological charge of spin angular momentum

Ref: K. Y. Bliokh, F. J. Rodríguez-Fortuño, F. Nori and A. V. Zayats. Nature Photonics 9: 796-808, 2015.

Teaching — Orbital Angular Momentum (4)

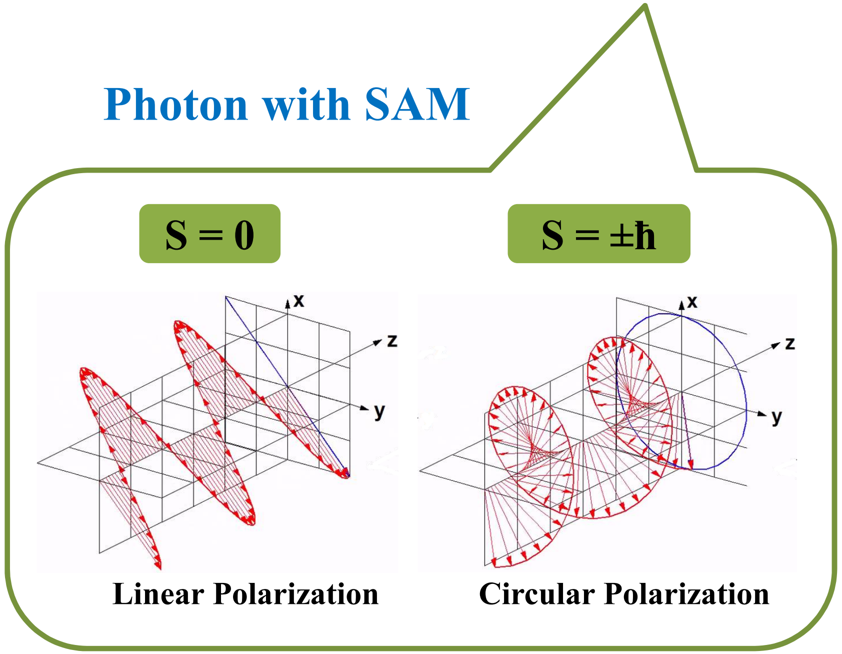
- **Spin angular momentum (SAM): value S = 0, ±ħ**
- **Orbital angular momentum (OAM): value L = *l*ħ**

trivial plane wave with zero topological charge

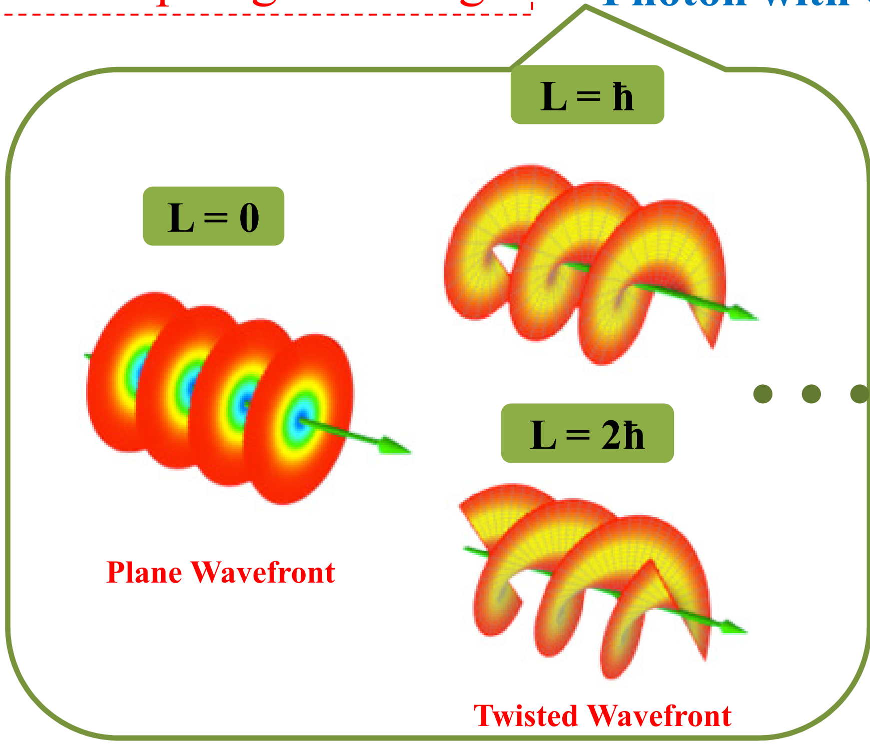


nontrivial vortex beam with nonzero topological charge

Photon with SAM



Photon with OAM



Teaching — Orbital Angular Momentum (5)

Even though many problems are a "done deal" in the physics community, they are not a "done deal" in an engineering community. And much blood, sweat, and tears are needed. —— By Prof. Weng Cho Chew

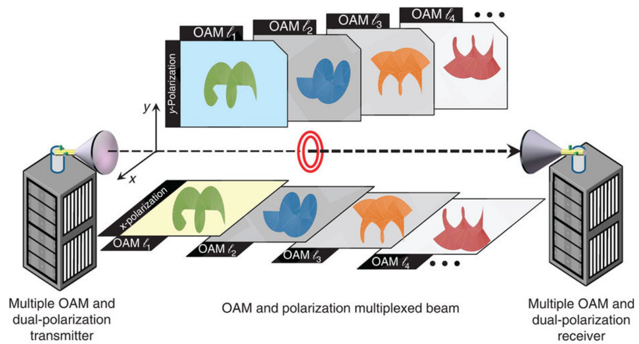
1. Limited scattering channels or the degrees of freedom (DoFs) bounded by the size D_t of a 2-D planar design of OAM ~ $(k_0D_t)^2$ [vortex beams cannot offer infinite number of communication channels]

2. Over-quadratic decay of the associated power density along the beam axis ~$z^{-2|l|-2}$

3. Turbulence influence [topologically robust?]

Reference:

1. Nature Photonics, 9: 822, 2015
2. IEEE Access, 6:19814, 2018



Nature Communications, 5: 4876, 2014

Learning — New Engineering Courses (PhD students)

Incomplete list….

Engineering ↔	**Physics** ↔	**Mathematics**
advanced EM theory	electrodynamics	numerical method
microwave engineering	thermodynamics	functional analysis
engineering EM	fluid dynamics	differential geometry
antenna theory	solid-state physics	group theory
signal processing	semiconductor physics	PDE
machine learning	quantum mechanics	statistics

Commercial Software

FEKO
HFSS
CST,
COMSOL
FDTD Solutions
RSoft
EastWave
…

Lack of compulsory training in CEM leads to :

- ✓ unexpectedly long simulation time
- ✓ huge consumption of computer resources
- ✓ incorrect modeling results
- ✓ unaware mistakes in concepts and designs

Examples for students:

Do u know the difference between MUMPS and PCG solvers in COMSOL?
Do u know how to truncate guided-wave structures (port v.s. PML)?
Do u know how to treat a dipole source in periodic structures (periodic BC)?
Do u know how to postprocess your data?
…

Commercial software cannot solve cutting-edge multiphysics problems!

Conclusion

1. As the progress of emerging technology and science, such as nanotechnology and quantum science, future engineers are to be exposed in circumstances where multidisciplinary research and development play more and more critical roles.

2. Educators have to teach EM knowledge with updated physical understandings and engineering applications in flexible delivery scenarios. The depth and breadth of EM knowledge should be enhanced.

3. New engineering course which merges mathematics, physics with engineering is an effective solution to tackle the challenges in future EM education.

4. Compulsory training in CEM is essential to understand and verify physical concepts and to design and optimize engineering applications.

Acknowledgement

Thanks for Your Attention!