Lasers

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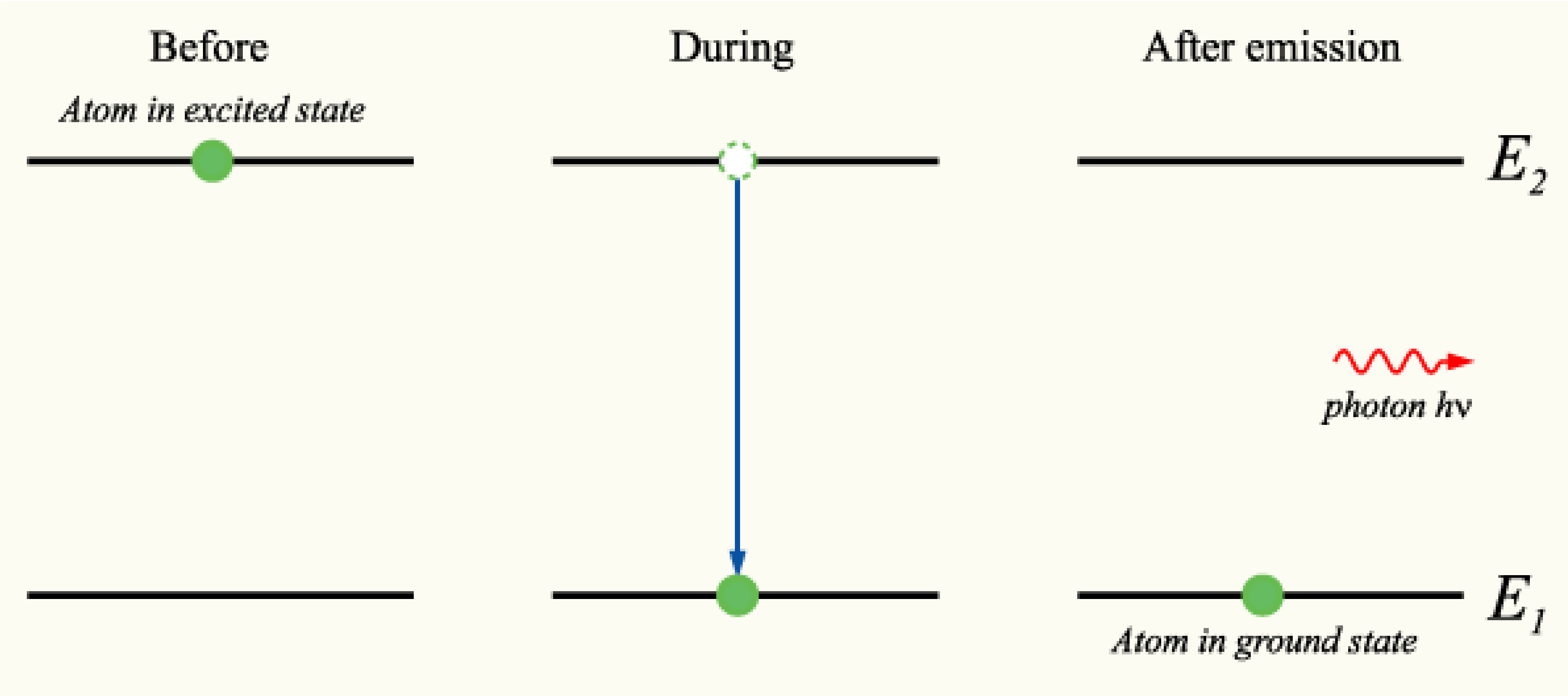
Course Overview

1. Spontaneous Emission and Stimulated Emission
2. Population Inversion and Three-Level Laser
3. Four-Level Laser and Rate Equations
4. Laser Modes
5. Broadening of Optical Gain
6. Pulsed Laser
7. Laser Diode
8. Heterostructure Laser Diodes
9. Quantum Well Laser
10. Single Frequency Lasers

Ref:
Kasap, Optoelectronics and Photonics (Ed. 2), Chapter 4;
Chuang, Physics of Photonic Devices, Chapter 10.

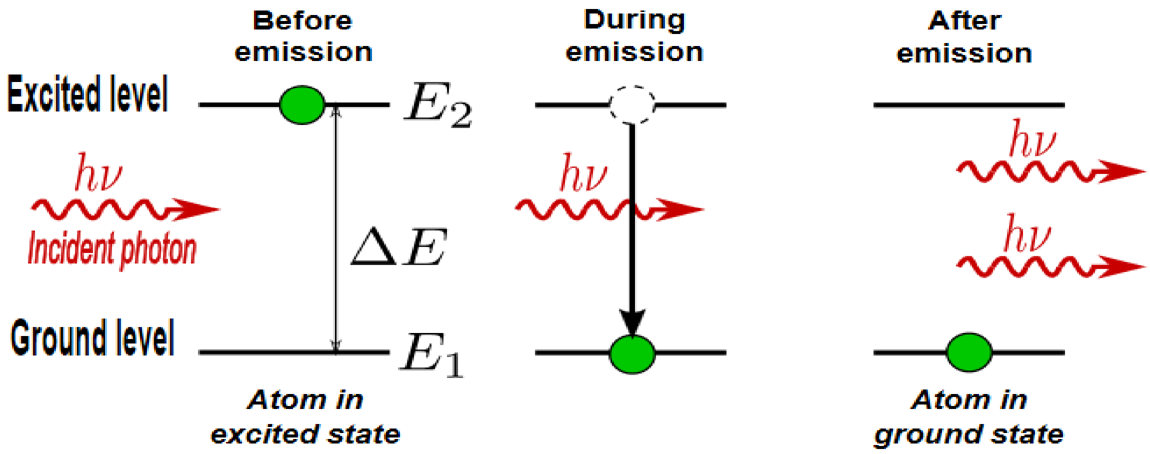
1. Spontaneous Emission and Stimulated Emission (1)



Spontaneous emission is the process where electron in an excited state undergoes a transition to a state with a lower energy (typically, the ground state) and emits a photon. It occurs at random intervals emitting incoherent, non-directional, and un-polarized photons.

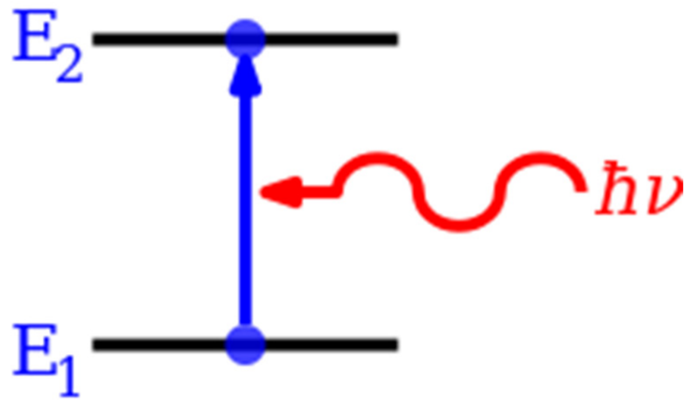
1. Spontaneous Emission and Stimulated Emission (2)

<u>simulated emission</u>



$$E_2 - E_1 = \Delta E = h\nu$$

<u>absorption</u>



Stimulated emission is the process where an incoming photon interacts with an excited electron, causing it to drop to a lower energy level. The emitted new photon with <u>identical phase, frequency, polarization, and direction</u> of travel as the photons of the incident wave.

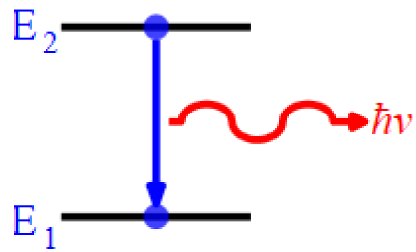
Absorption is the process where a photon is absorbed by an electron, causing the electron to jump from a lower energy level to a higher one. The absorption can be seen as a reverse process of stimulated emission.

1. Spontaneous Emission and Stimulated Emission (3)

Einstein coefficients are mathematical quantities which are a measure of the probability of absorption or emission of light. The Einstein *A* coefficient is related to the rate of spontaneous emission of light and the Einstein *B* coefficients are related to the absorption and stimulated emission of light.

If n_i is electron density at state i , then the change of electron density in state 2 per unit time due to spontaneous emission will be:

$$\left(\frac{dn_2}{dt}\right)_{spon} = -A_{21}n_2$$



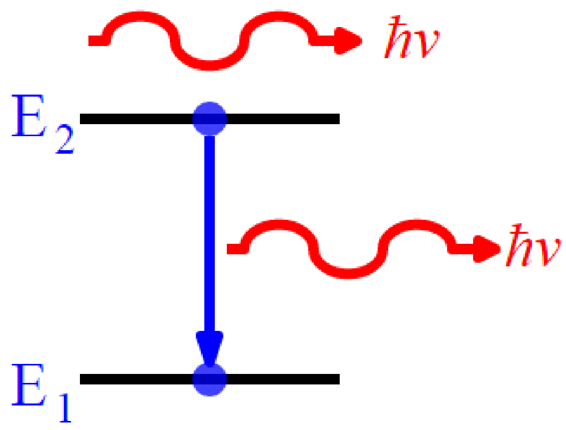
The same process results in increasing of the population of the state 1

$$\left(\frac{dn_1}{dt}\right)_{spon} = A_{21}n_2$$

1. Spontaneous Emission and Stimulated Emission (4)

The stimulated process is described by the Einstein coefficient B_{21} ($J^{-1}\ m^3\ s^{-2}$), which gives the probability per unit time per unit spectral energy density of the radiation field that an electron in state 2 with energy E_2 will decay to state 1 with energy E_1. The change in electron density at state 1 per unit time due to induced emission will be

$$\left(\frac{dn_1}{dt}\right)_{stim} = B_{21} n_2 \rho(v)$$

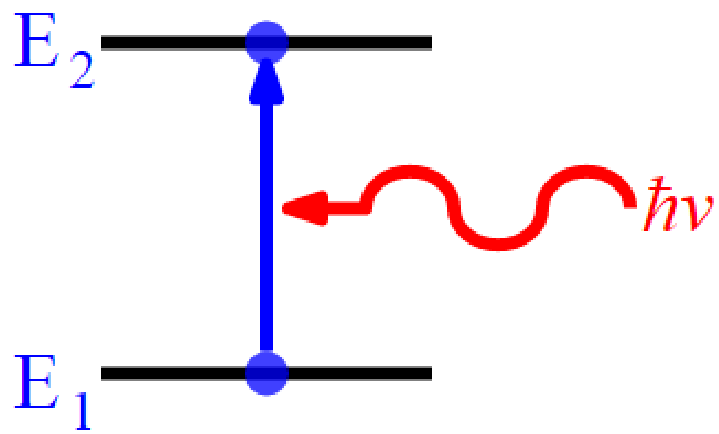


where $\rho(v)$ denotes the spectral energy density (energy per unit frequency and per unit volume) of the isotropic radiation field at the frequency of the transition (that can be obtained by black-body radiation law).

1. Spontaneous Emission and Stimulated Emission (5)

The absorption process is also described by the Einstein coefficient B_{12} ($J^{-1}\ m^3\ s^{-2}$), which gives the probability per unit time per unit spectral energy density of the radiation field that an electron in state 1 with energy E_1 will absorb a photon hv and jump to state 2 with energy E_2. The change in electron density at state 1 per unit time due to absorption will be:

$$\left(\frac{dn_1}{dt}\right)_{abs} = -B_{12} n_1 \rho(v)$$



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1. Spontaneous Emission and Stimulated Emission (6)

At thermodynamic equilibrium, the net change in the number of any excited electrons is zero, being balanced by loss and gain due to all processes. We will have detailed balancing as well, which states that the net exchange between any two levels will be balanced. Detailed balance requires that the net change of the number of electrons in level 1 due to the three processes is zero:

$$\boxed{A_{21}n_2 + B_{21}n_2\rho(v) - B_{12}n_1\rho(v) = 0}$$

Also, at thermal equilibrium, electron densities of the states 1 and 2 satisfy Boltzmann statistics

$$\boxed{\frac{n_2}{n_1} = \exp\left(-\frac{(E_2 - E_1)}{k_B T}\right)}$$

1. Spontaneous Emission and Stimulated Emission (7)

The relations of Einstein coefficients A and B

$$\boxed{\frac{A_{21}}{B_{21}} = \frac{8\pi h v^3}{c^3}} \qquad \boxed{B_{12} = B_{21}}$$

Assignment hints:

1. Black-body radiation energy per unit frequency and per unit volume

$$\rho(\mathrm{v}) = \frac{8\pi h v^3}{c^3} \frac{1}{e^{hv/\mathrm{k}_B T} - 1}$$

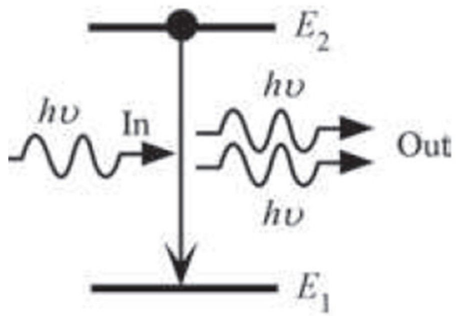
2. Detailed balance equation is valid at arbitrary temperature T

2. Population Inversion and Three-Level Laser (1)

When we are considering a collection of atoms to amplify light, we must therefore have the majority of the atoms at the energy level E_2. If this were not the case, the incoming photons would be absorbed by the atoms at E_1.

When there are more atoms at E_2 than at E_1, we have what is called a **population inversion**. Under normal equilibrium conditions, as a result of Boltzmann statistics, most of the atoms would be at E_1, and very few at E_2. With only two energy levels, we can never achieve the population inversion. We need at least three energy levels!

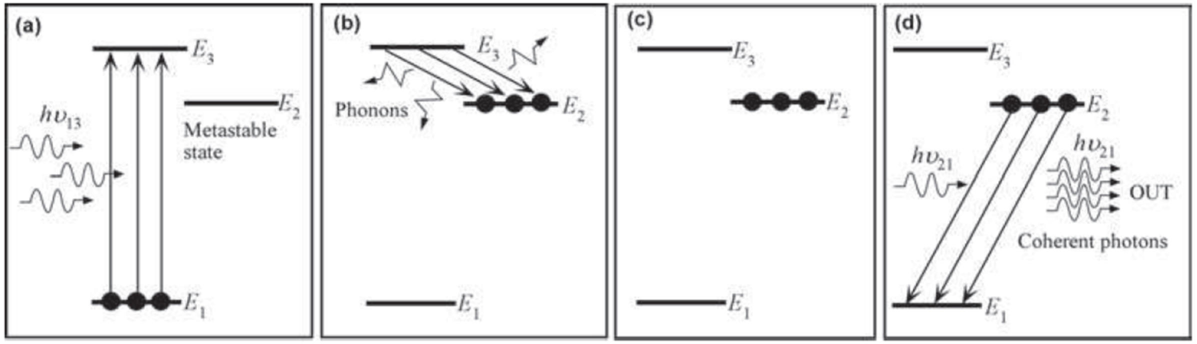
Once more than half the electrons/ions at E_1 have been pumped, there is population inversion and consequently the pumped medium exhibits optical gain, or photon amplification. The emission from E_2 to E_1 is called the lasing emission. The system for photon amplification is a laser, an acronym for Light Amplification by Stimulated Emission of Radiation.



Stimulated emission

2. Population Inversion and Three-Level Laser (2)

Three-level laser



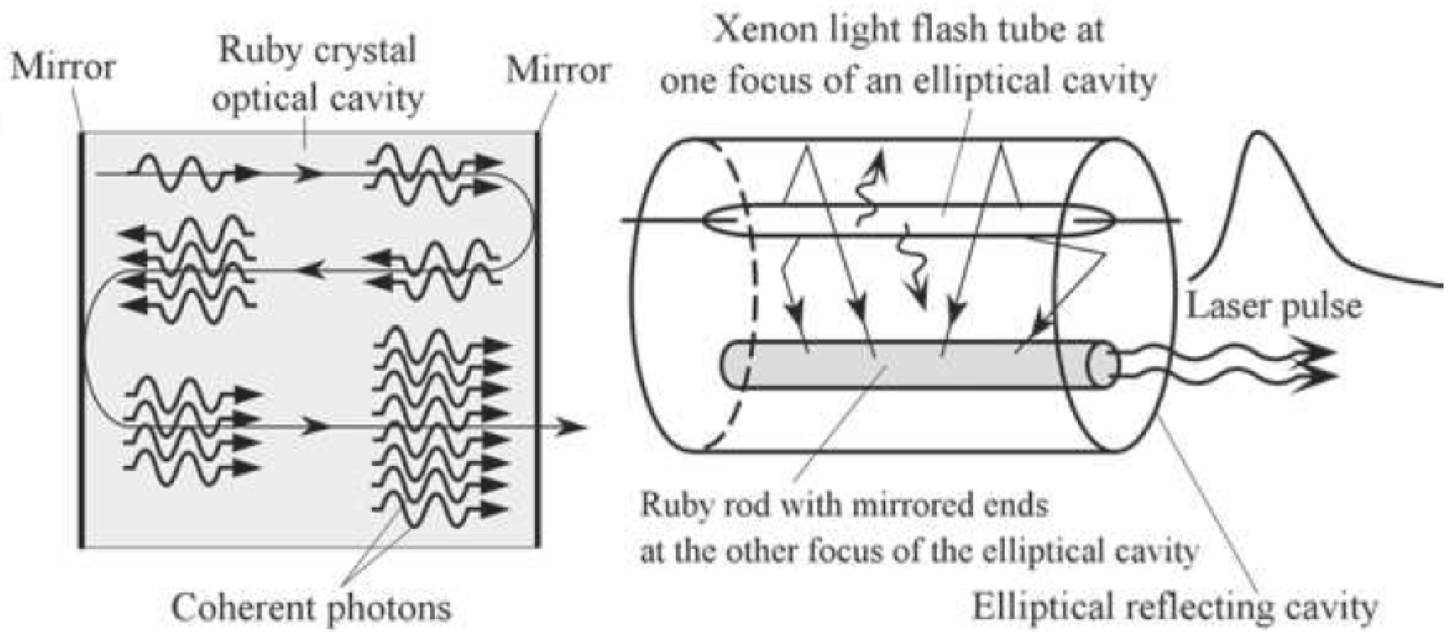
E_1, E_2, and E_3 correspond to the energies of chromium ions (Cr_{3+}) in a crystal of Al_2O_3.

E_1 and E_3 are the ground energy and pump energy levels. The process of exciting the ions from E_1 to E_3 is called optical pumping. From E_3, the ions decay rapidly to an energy level E_2 by emitting phonons (radiationless). **The energy level E_2 is a state that does not rapidly and spontaneously decay to the lower-energy state E_1**. In other words, the state at E_2 is a long-lived state (~ms).

With ample pumping, we can accumulate sufficient ions at E_2 to cause a population inversion, as pumping takes more and more ions to E_3 and hence E_2. The spontaneously emitted photon can go on to a neighboring ion and executes stimulated emission. The photons from the latter can go on to the next ion at E_2 and cause that to emit by stimulated emission and so on. The result is an avalanche of stimulated emission processes with all the photons in phase, in the same direction, and with the same polarization (emitted photons are coherent).

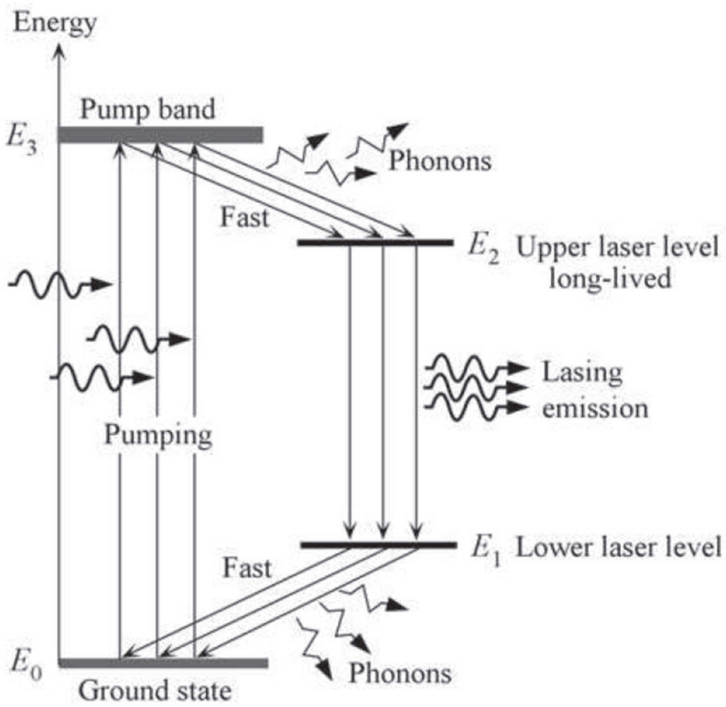
Optical cavity

To get coherent lasing radiation out from the ruby laser, we need to do more, not just amplify the radiation along the ruby crystal. We can increase stimulated emissions by increasing the number of photons, *i.e.,* the radiation intensity. The ends of the ruby crystal, which is normally a rod, are silvered to reflect back and forward the stimulated radiation, that is, to form an **optical cavity** with mirrors at the ends.



3. Four-Level Laser and Rate Equations (1)

Four-level laser



The problem with the three-level laser system is that to achieve population inversion, we must pump at least half the Cr_{3+} ions at the ground level E_1 to $E_{2.}$ We can quickly and easily achieve population inversion if we use a four-level system.

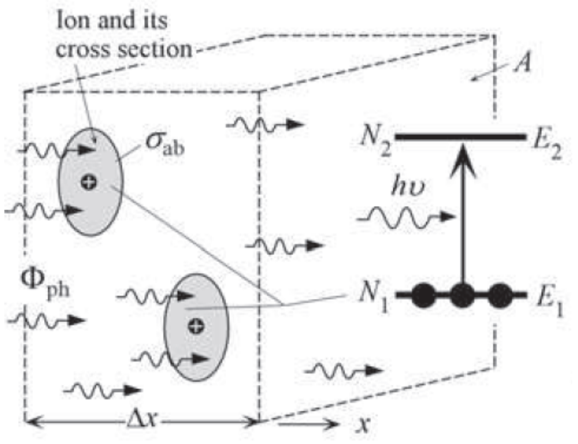
All the ions are at E_0 whereas E_1, E_2, and E_3 are mostly empty. From E_3, the ions decay rapidly by phonon emission to E_2, which is a long-lived state with a lifetime in the milliseconds. Clearly, as soon as ions begin to populate E_2, we have population inversion between the level E_1 and E_2 because E_1 is initially nearly empty.

The lasing emission takes place by stimulated emission as ions drop from E_2 to E_1. From E_1, the ions return back to E_0 by the emission of phonons. We also need the ions at E_1 to decay quickly back to E_0 so that a buildup of ions at E_1 is avoided, and the ions are quickly returned back to E_0 for repumping. Most modern lasers are based on a four-level system (Nd^{3+}: YAG ($Y_3Al_5O_{15}$) 掺钕钇铝石榴石 laser).

Emission and absorption cross-sections

The interaction of a photon with an individual ion can be intuitively visualized as the ion having a certain *cross-sectional area*. Optical power absorbed by an ion is equal to light intensity multiplied by absorption cross-section of ion

$$P_{ab} = I\sigma_{ab}$$



Absorption

Consider a small volume $A\Delta x$ of the medium, the total optical power absorbed in this volume is then

$$\Delta P_{ab} = I\sigma_{ab}N_1\left(A\Delta x\right)$$

where N_1 is the number of ions at E_1.

Suppose that ΔI is the change in the intensity

$$A\Delta I = -I\sigma_{ab}N_1\left(A\Delta x\right)$$

fractional change in the light intensity per unit distance

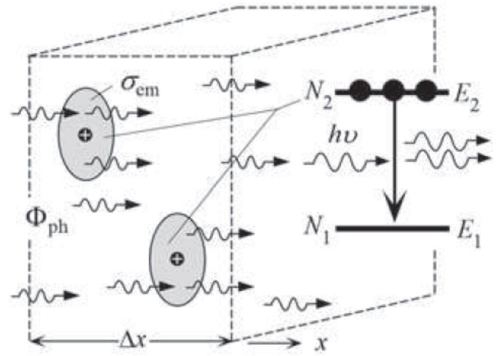
$$\boxed{-\frac{\Delta I}{I\Delta x} = \sigma_{ab}N_1 = \alpha}$$

→ absorption coefficient

3. Four-Level Laser and Rate Equations (3)

Stimulated optical power emitted by an ion is equal to light intensity multiplied by emission cross-section of ion

$$P_{em} = I\sigma_{em}$$

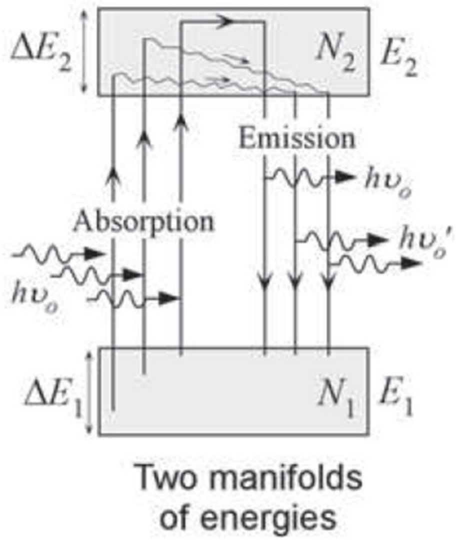


Similarly, the fractional increase in the intensity per unit distance

$$\boxed{\frac{\Delta I}{I\Delta x} = \sigma_{em} N_2}$$

The net fractional increase in the intensity per unit distance is defined as the **gain coefficient** g of the medium

$$\boxed{g(v) = \left(\frac{\Delta I}{I\Delta x}\right)_{net} = \sigma_{em}(v)N_2 - \sigma_{ab}(v)N_1}$$



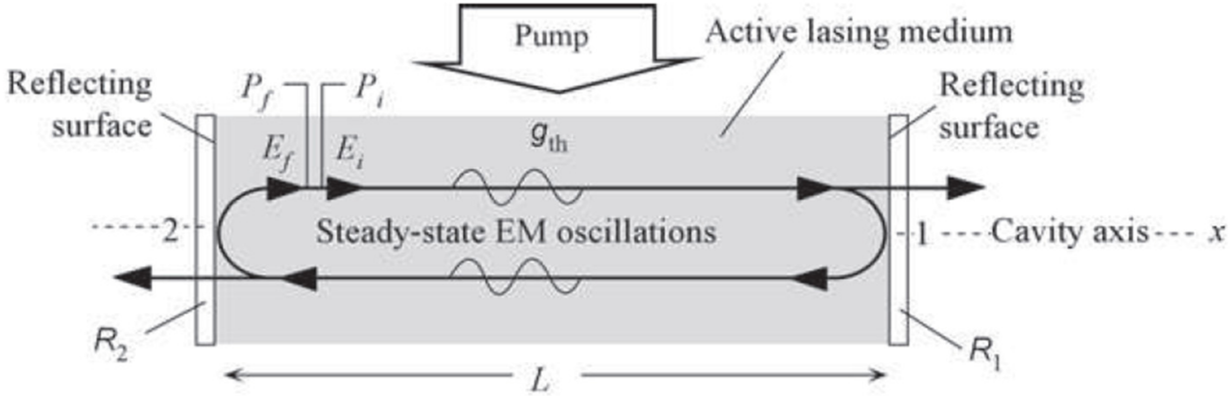
overall optical power gain G through a medium of length L

$$\boxed{G = \exp(gL)}$$

3. Four-Level Laser and Rate Equations (4)

Lasing conditions

1. Population inversion
2. Optical cavity
3. Gain is larger than loss



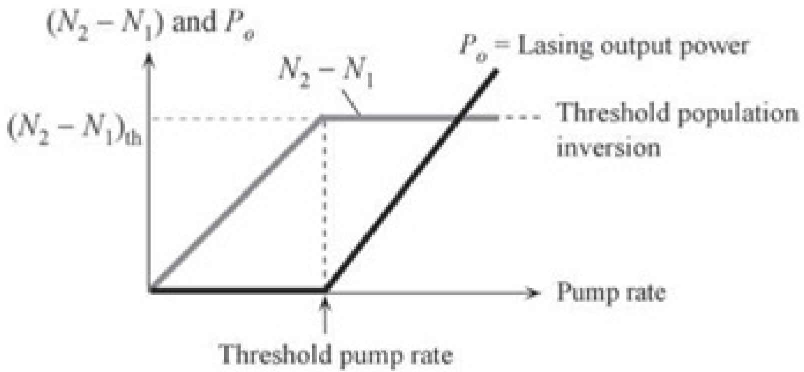
Power P_f of the EM radiation after one round trip of path length $2L$

$$P_f = R_1 R_2 \exp(-\alpha_s 2L) \exp(g 2L) P_i$$

α_s is the loss by cavity wall

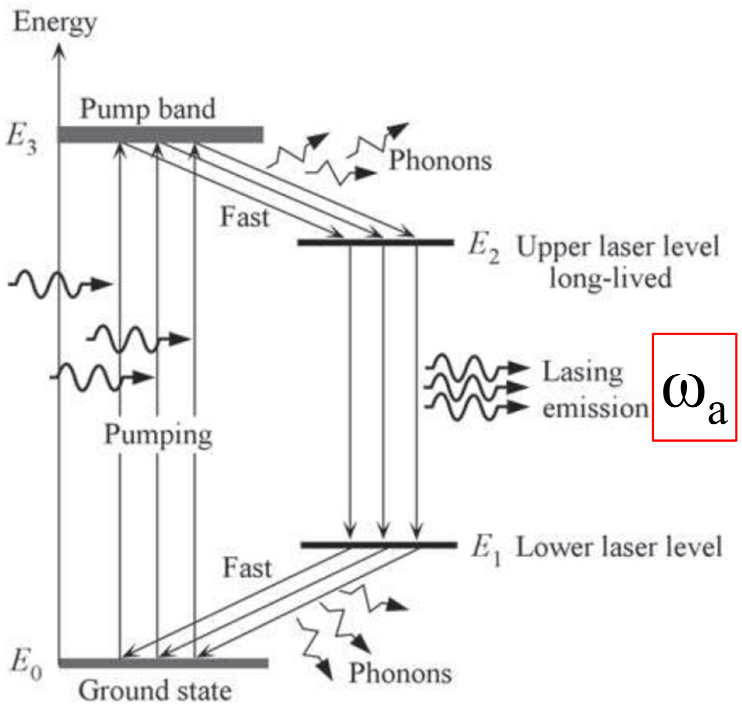
Threshold gain coefficient is obtained by making $P_f / P_i = 1$

$$g_{th} = \alpha_s + \frac{1}{2L} \ln\left(\frac{1}{R_1 R_2}\right)$$



Initially the medium must have a gain coefficient g greater than g_{th}. This allows the oscillations to build up in the cavity until **a steady state is reached when $g = g_{th}$**.

3. Four-Level Laser and Rate Equations (5) (Optional)



$$\frac{\partial N_3(r,t)}{\partial t} = P_g(t)N_0(r,t) - \frac{N_3(r,t)}{\tau_{32}}$$

$$\frac{\partial N_2(r,t)}{\partial t} = \frac{N_3(r,t)}{\tau_{32}} + \frac{1}{\hbar\omega_a}\mathbf{E}(r,t)\cdot\frac{\partial \mathbf{P}(r,t)}{\partial t} - \frac{N_2(r,t)}{\tau_{12}}$$

$$\frac{\partial N_1(r,t)}{\partial t} = \frac{N_2(r,t)}{\tau_{12}} - \frac{1}{\hbar\omega_a}\mathbf{E}(r,t)\cdot\frac{\partial \mathbf{P}(r,t)}{\partial t} - \frac{N_1(r,t)}{\tau_{10}}$$

$$\frac{\partial N_0(r,t)}{\partial t} = \frac{N_1(r,t)}{\tau_{10}} - P_g(t)N_0(r,t)$$

pumping rate

coupling strength

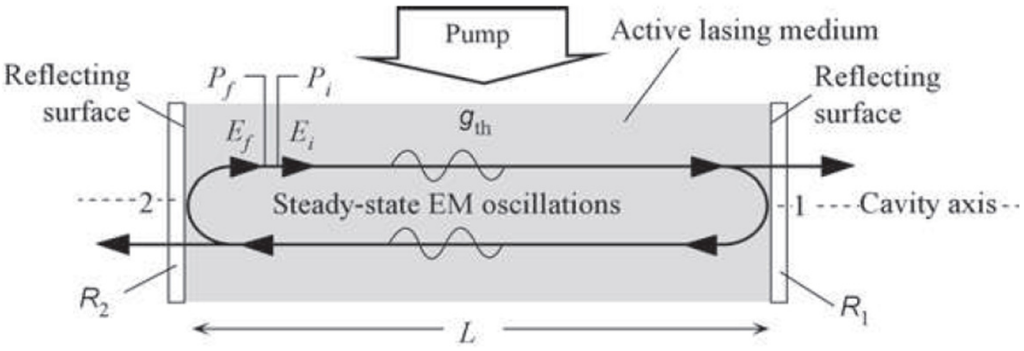
$$\frac{\partial^2 \mathbf{P}(r,t)}{\partial t^2} + \Gamma_a\frac{\partial \mathbf{P}(r,t)}{\partial t} + \omega_a^2\mathbf{P}(r,t) = \sigma_a \Delta N(r,t)\mathbf{E}(r,t)$$

$$\Delta N(r,t) = N_1(r,t) - N_2(r,t)$$

Assignment: How to connect pumping rate with pump intensity?

Phys. Rev. Lett. 108: 187402, 2012

4. Laser Mode



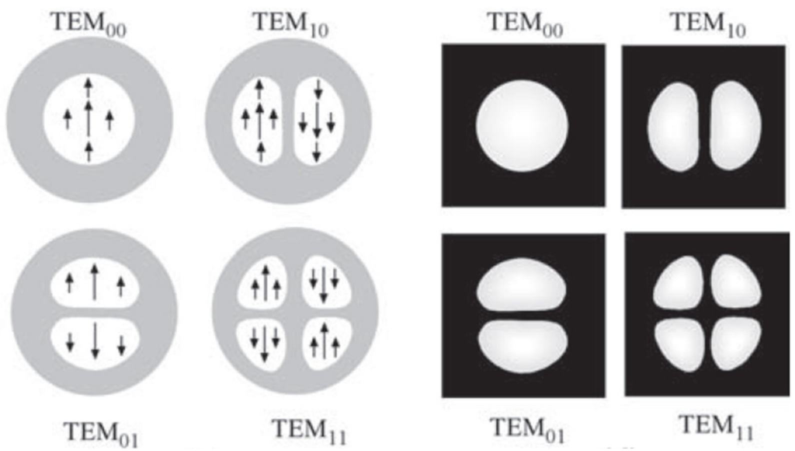
If ignoring the phase by reflection, resonance condition for length L is

$$2L\left(k_m n\right) = m2\pi$$

$$\boxed{L = m\frac{\lambda_m}{2n}}$$

the modes above are controlled by the length L of the optical cavity along its axis and are called **longitudinal axial modes**.

More generally, whether we have flat or spherical end mirrors, we can find all possible allowed modes by considering what spatial field patterns at the mirrors



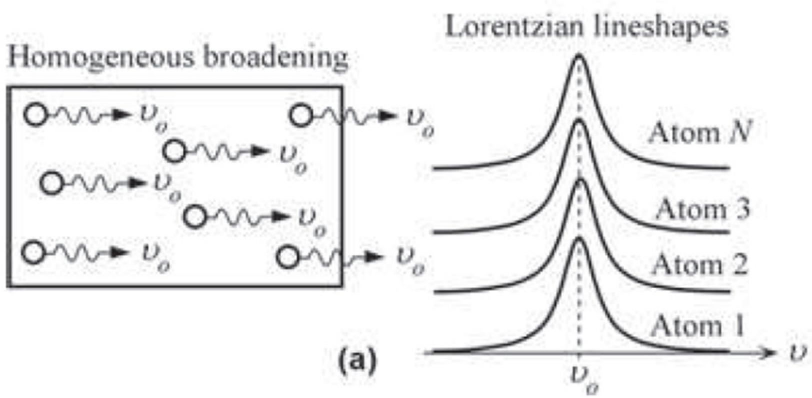
they are referred to as transverse modes or **transverse electric and magnetic (TEM) modes.**

For TEM_{pqm} mode, the integer m is the usual longitudinal mode number (very large ~10^6).

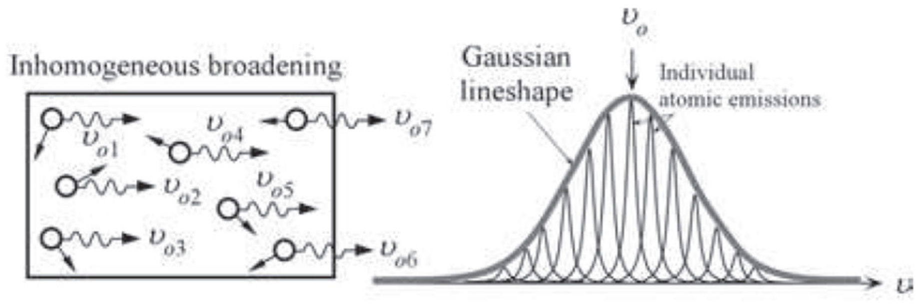
5. Broadening of Optical Gain (1)

Emission spectrum involves transitions from a small region ΔE_2 around E_2 to a small region ΔE_1 around E_1, and consequently has a finite spectral width determined by life time τ_2 and τ_1 (uncertainty principle). This type of broadening of the emission spectrum, and hence the optical gain curve (its lineshape), is called **lifetime broadening**. This type of spectral broadening in which all the atoms in a medium generate the same emission curve with the same central frequency and subjected to the same broadening mechanism is called **homogeneous broadening**. The shape of the emission curve is called a **Lorentzian lineshape**.

Phonon collisions suddenly change the phase of the EM wave during emission. This type of broadening mechanism is called **collision broadening (pressure broadening** for gas), which is also homogeneous broadening.



5. Broadening of Optical Gain (2)



(b)

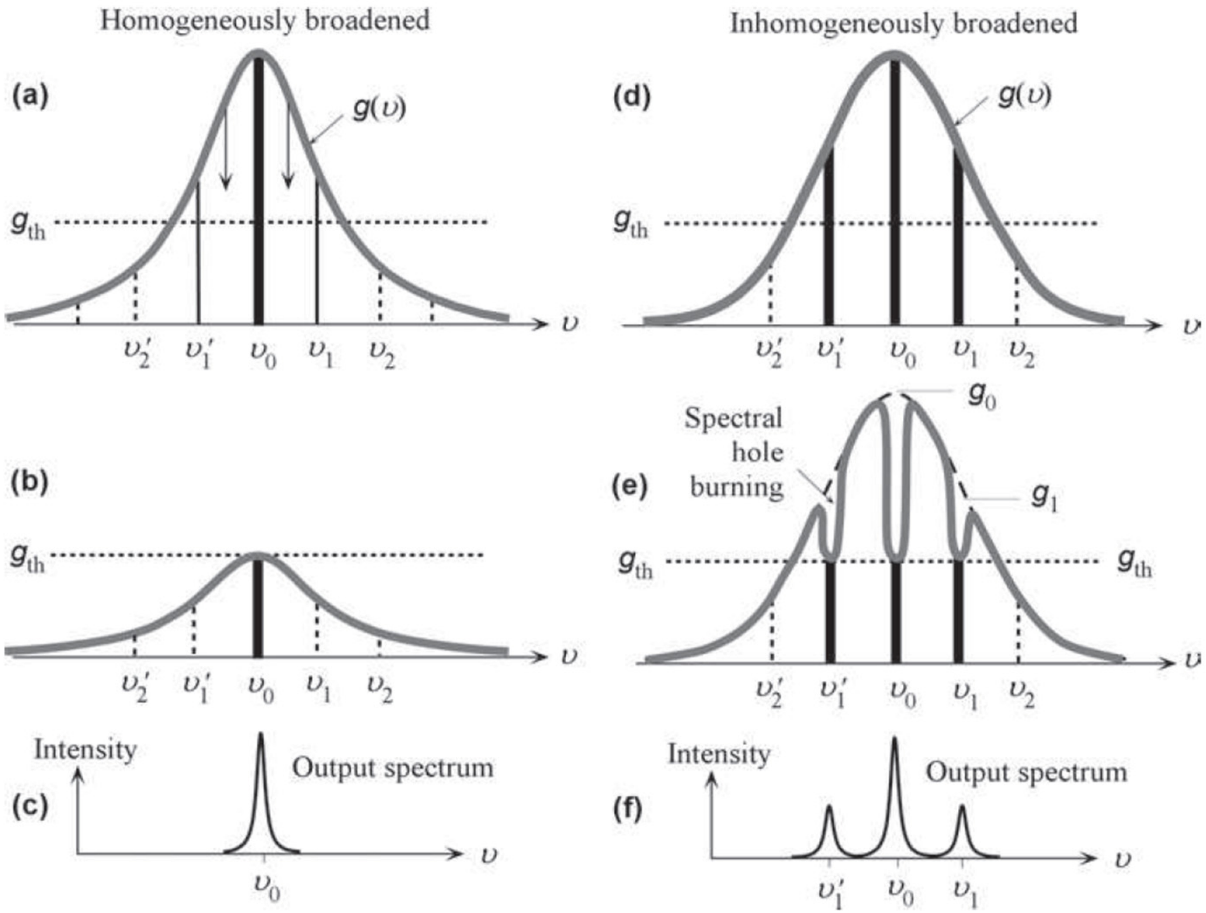
$$\upsilon_1 = \upsilon_o\left(1 - \frac{v_x}{c}\right) \qquad \upsilon_2 = \upsilon_o\left(1 + \frac{v_x}{c}\right)$$

In the He-Ne gas laser, individual Ne atoms (moving randomly) emitted at different frequencies due to the **Doppler effect**. The Doppler effect shifts the emission curve of each Ne atom an amount that depends on the velocity of the Ne atom and its direction.

With a large number of Ne atoms, these different randomly shifted emission curves add to produce an overall emission lineshape that is a **Gaussian**. This type of broadening in which each atom emits at a slightly different central frequency is called **inhomogeneous broadening**.

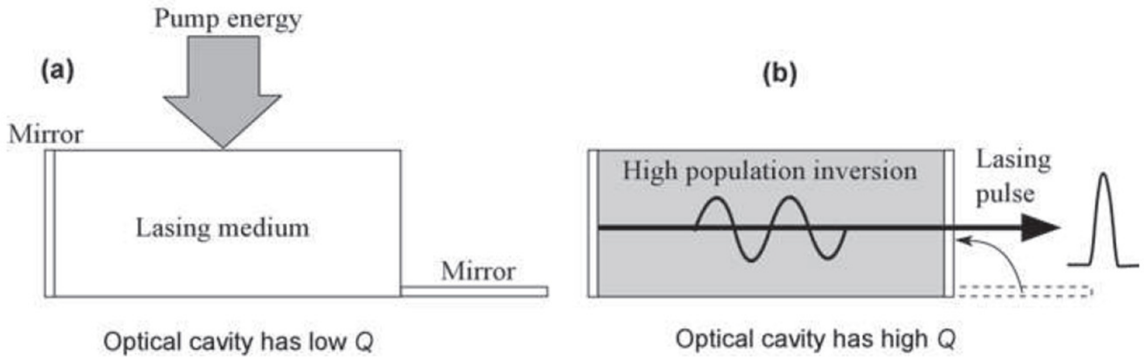
Inhomogeneous broadening also occurs in solid state glass lasers, for example, in Nd^{3+} glass lasers. Within the glass structure, the Nd^{3+} ions find themselves in different local environments, with different neighboring ions. These variations in the local environment from site to site in the glass result in the energy levels of the Nd^{3+} ion becoming spread. This type of inhomogeneous broadening is often termed **amorphous structure broadening**.

5. Broadening of Optical Gain (3)



Assignment: why does **spectral hole burning** of gain occur for inhomogeneous broadening case?

6. Pulsed Laser (1)

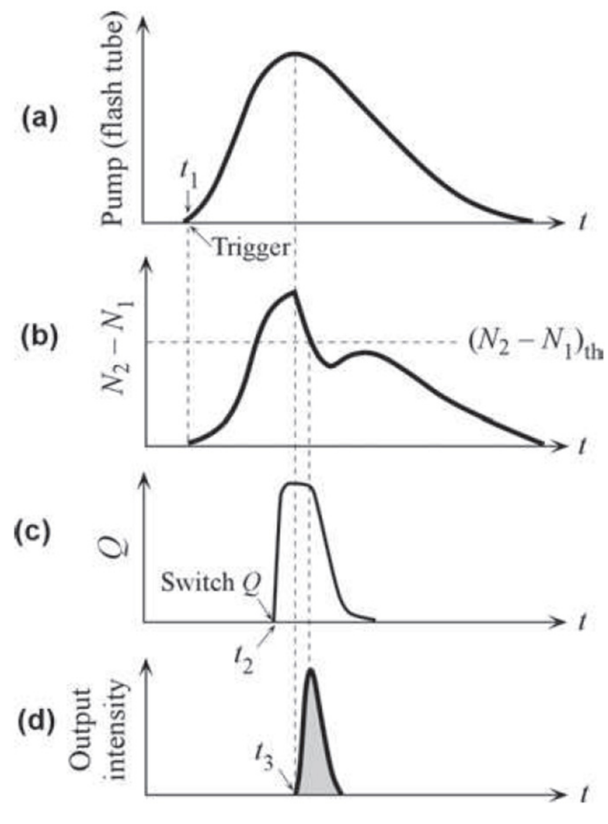


t1~t2 quickly build up N_2-N_1 above the threshold value
t2~t3 build up lasing radiation
t3 ~ lasing transitions/pulse forming

***Q*-switching** refers to a laser whose optical resonant cavity is switched from a low *Q* to a high *Q* to generate an intense laser pulse.

If cavity has low *Q*, the lasing is suppressed. Without the reflections from the mirrors, the photon energy density in the cavity is small, and the active medium can be pumped to achieve a large population inversion.

As soon as the *Q* is switched to a high value, the low loss in the optical resonator allows lasing oscillations to occur, which deplete the population inversion (de-excite atoms) and decrease the gain until the population inversion falls below the threshold value and lasing oscillations cease.



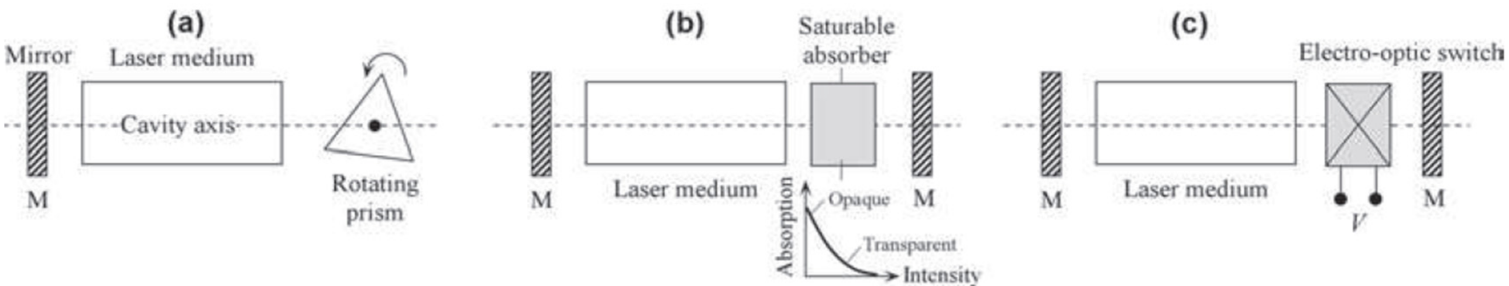
Q-switching time
photon cavity lifetime

Physical implementation of *Q*-switching

1. The rotation of the prism every time a prism face aligns with the cavity axis

2. Another technique is to use a saturable absorber in the optical cavity. A saturable absorber's absorption decreases with increasing light intensity so that it becomes transparent only at high intensities. Absorber becomes transparent, which corresponds to switching the cavity *Q* to a high value.

3. Use of an electro-optic (EO) switch in the optical cavity could achieve *Q*-switching as well. EO switch (very fast, less than nanoseconds) is essentially an electro-optic (or a piezoelectric) crystal with electrodes, and works in combination with a polarizer. EO switch is turned transparent by the application of a voltage, the cavity *Q* is switched high.

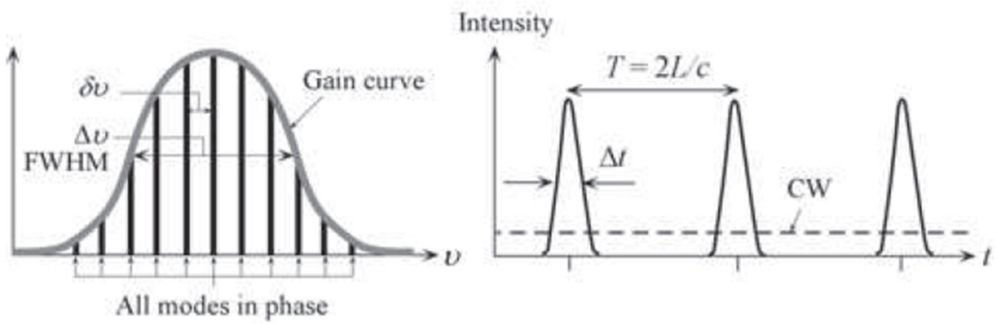


6. Pulsed Laser (3)

Mode locking is used to generate short and intense laser light pulses at a certain repetition rate that depends on the laser construction. A **mode-locked laser** is a laser that has been constructed to have one transverse mode and many (*N*) longitudinal modes that have the same phase.

Normally, the longitudinal modes of a laser cavity would be "independent" with random relative phases. In such a case, the output intensity from the laser is the sum of the individual mode intensities, i.e., $(I_1 + I_2 + .. + I_N)$. If the modes have been forced to have the same phase, then these mode oscillations would reinforce each other, they generate an intense optical pulse at a certain repetition rate. In this mode-locked case, the intensity in a pulse is proportional to $(E_1 + E_2 + .. + E_N)^2$.

When modes have been locked, there must be an optical pulse in the resonator traveling between the mirrors with exactly the required round-trip time of $T = 2L/c$.



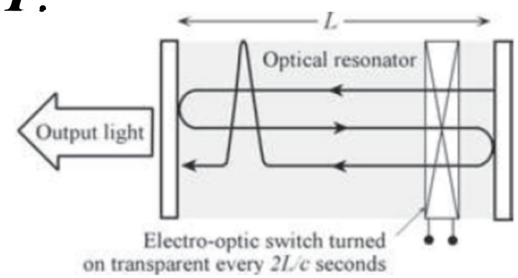
Assignment: How to calculate frequency width of laser pulse?

6. Pulsed Laser (4)

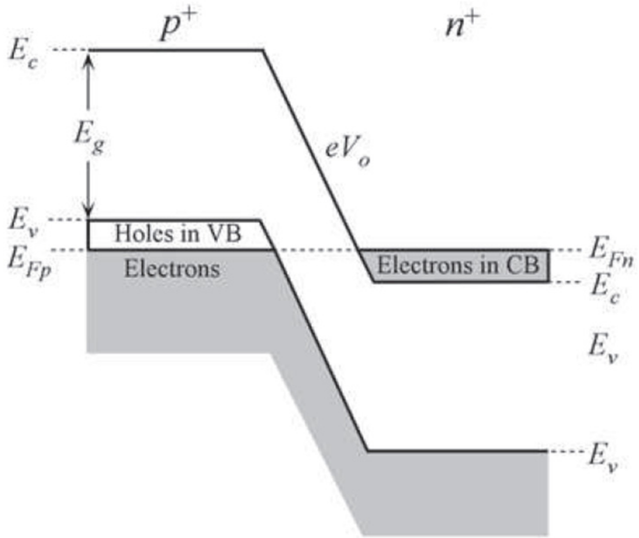
Physical implementation of *mode-locked laser*

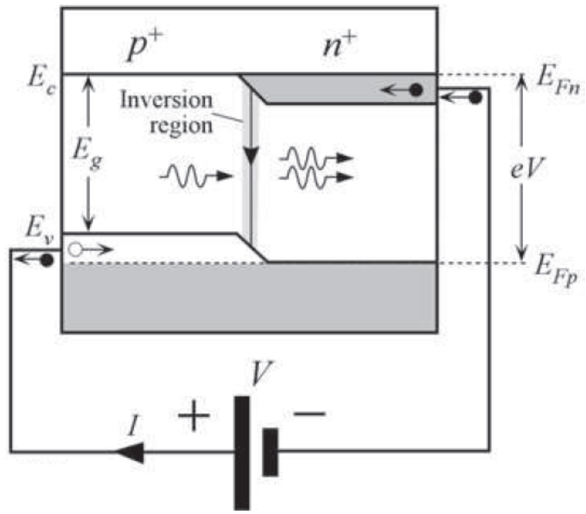
Suppose that we insert an *electro-optic switch* that is switched to be made transparent at every T seconds (as we did for Q-switching). The switch is on to be transparent only when the pulse is there and only for the duration of the pulse. This is an example of **active mode locking**. The difference from the electro-optically Q-switched laser is that in mode locking, the EO switch must be **turned on every *T* seconds for a very short time**, so the timing in sequence is critical.

Another possibility is **passive mode locking** where a saturable absorber is used in the optical cavity (again, similar to Q-switching). The optical cavity is lossy for low light intensities. Such an absorber would only allow a high intensity light pulse to exist in the cavity since it is only transparent at high intensities. The latter would correspond to various modes having the right phases to yield an intense pulse, that is, mode locking. The difference from the Q-switching case is that the absorber in a mode-locked laser must be able to **respond faster than the time *T***.



7. Laser Diode (1)





$$\boxed{E_{Fn} - E_{Fp} = eV > E_g}$$

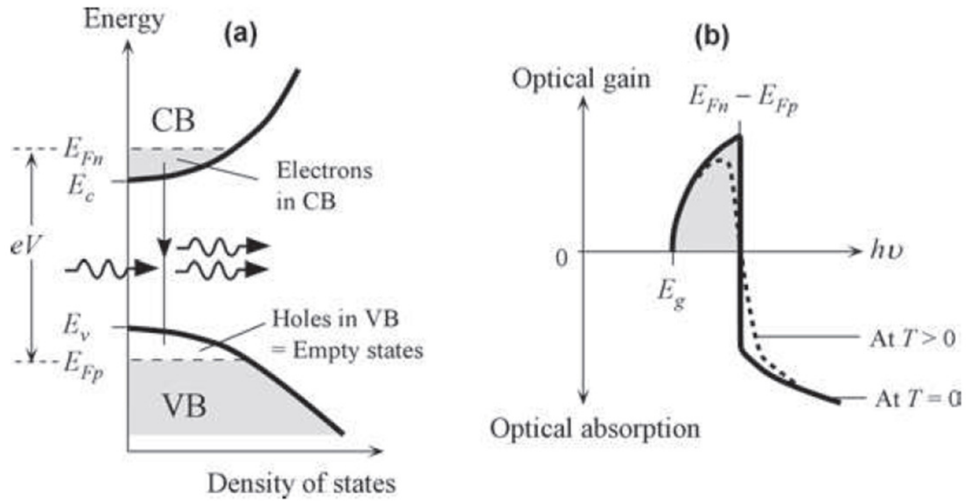
By **degenerate doping** we mean that the Fermi level E_{Fp} in the *p*-side is in the valence band (VB) and E_{Fn} in the *n*-side is in the conduction band (CB).

The depletion region or the space charge layer (SCL) in such a pn junction is very narrow.

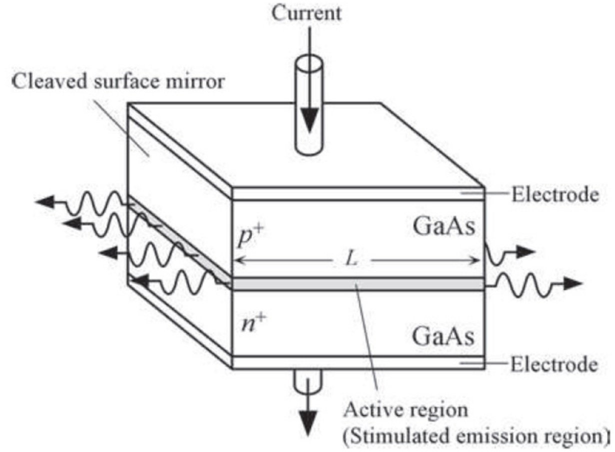
This degenerately doped pn junction is **forward biased** by a voltage V greater than the bandgap Voltage $\boldsymbol{eV > E_g}$. Electrons from n+ side and holes from p+ side flow into the SCL, and this **SCL region is no longer depleted**.

In this region, there are more electrons in the conduction band at energies near E_c than electrons in the valence band near E_v. In other words, there is a **population inversion** between energies near E_c and those near E_v around the junction.

7. Laser Diode (2)



homojunction laser diode

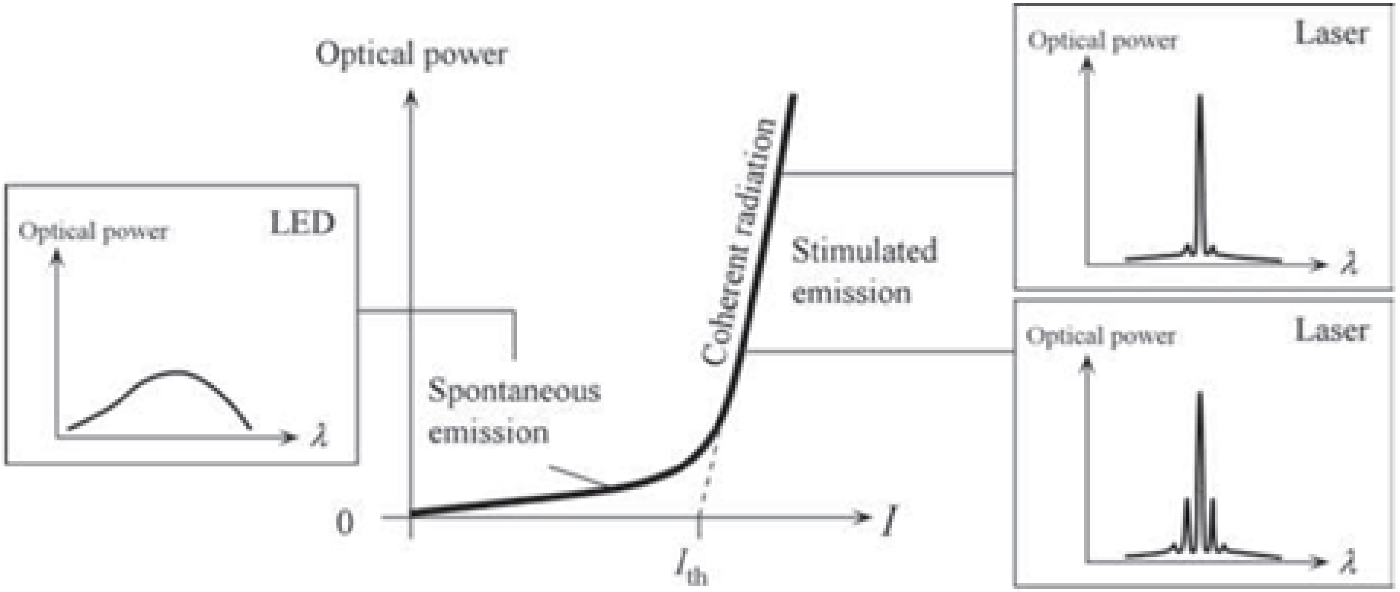


This population inversion region is a layer along the junction and is called the **inversion layer** or the **active region**. **Photons with energy greater than E_g but less than E_{Fn} - E_{Fp} cause stimulated emissions**, whereas those photons with energies greater than E_{Fn} - E_{Fp} become absorbed.

The pumping mechanism is therefore the forward diode current and the pumping energy is supplied by the external battery. This type of pumping is called <u>injection pumping</u>.

In addition to population inversion we also need to have an optical cavity to implement a laser oscillator.

7. Laser Diode (3)

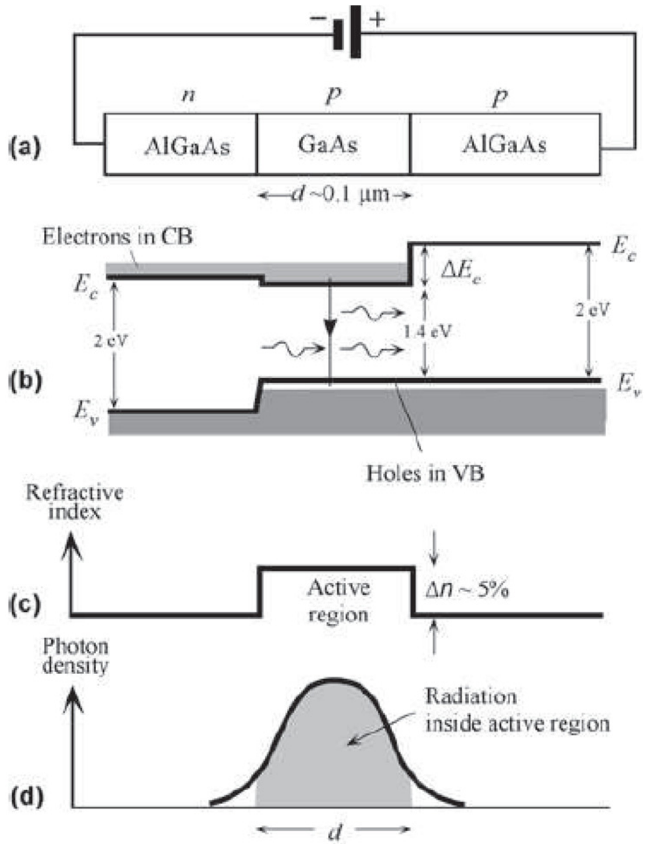


Below I_{th}, the light from the device is due to spontaneous emission and not stimulated emission. The light output is then composed of incoherent photons that are emitted randomly and the device behaves like an LED. For diode currents above I_{th}, the device emits coherent lasing emission. Heterostructure Laser has much smaller <u>**threshold current**</u> than homojunction laser.

The reduction of the threshold current:

- confine the injected electrons and holes to a narrow region around the junction
- build a dielectric waveguide around gain region to increase photon concentration

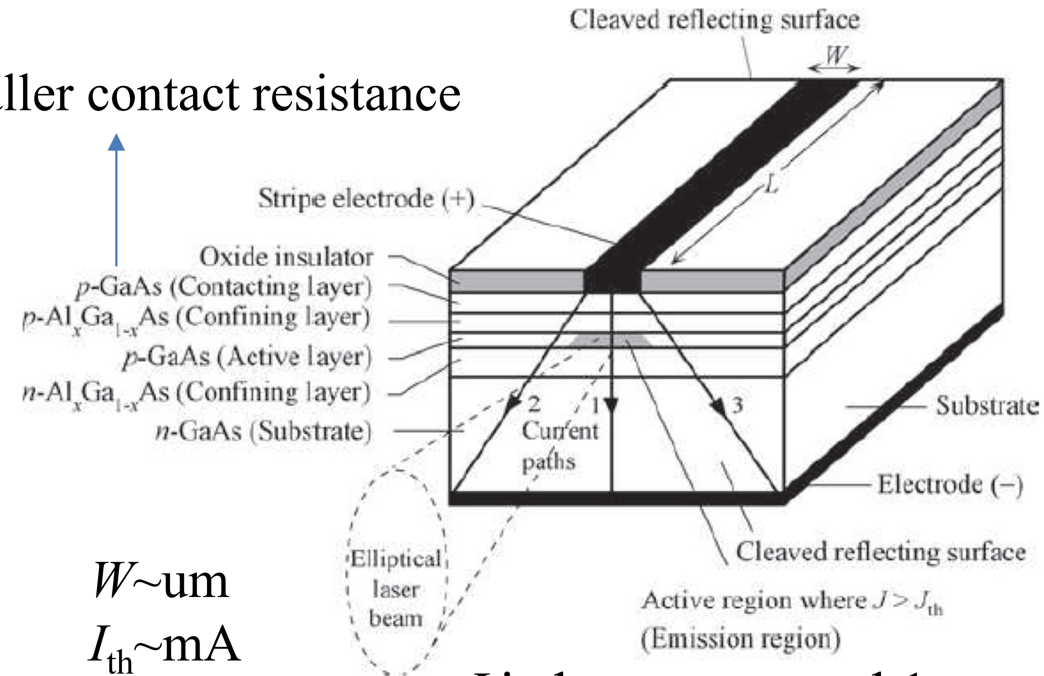
We therefore need both **carrier confinement** and **photon** or **optical confinement**



reduced contact area reduces the threshold current I_{th}.

reduced emission area makes light coupling to optical fibers easier.

smaller contact resistance



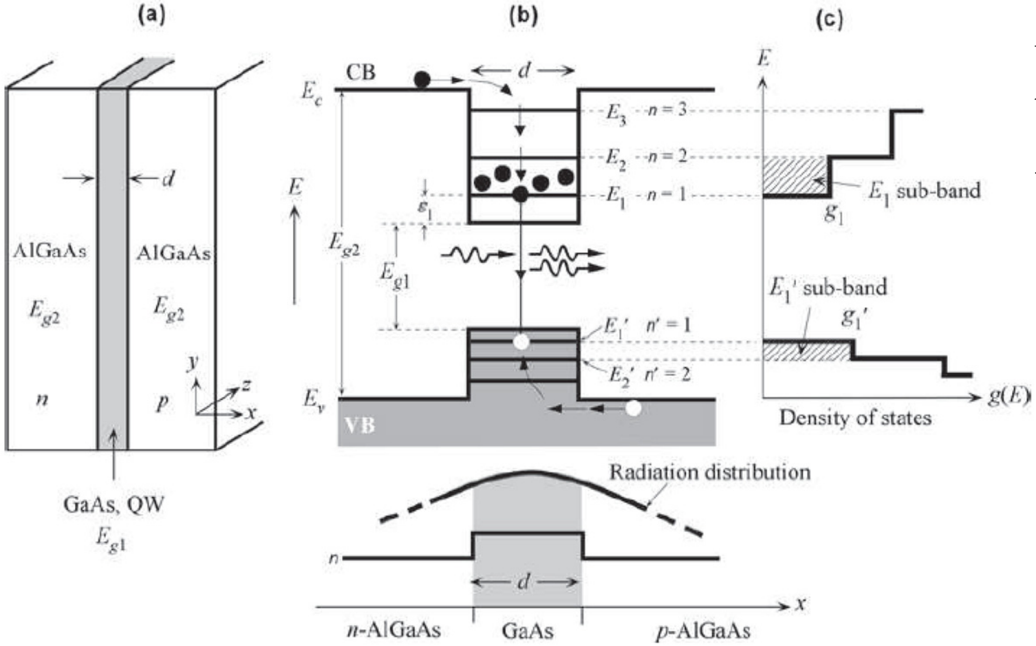
W~um

I_{th}~mA

J is largest around 1

9. Quantum Well Laser

Alternative way to reduce the threshold current



ultrathin narrow bandgap GaAs sandwiched between two wider bandgap semiconductors, which results in a QW (quantum well).

The energy of the electron in the QW is quantized along the *x*-direction of confinement.

The electrons in the conduction band form a 2D free electron gas.

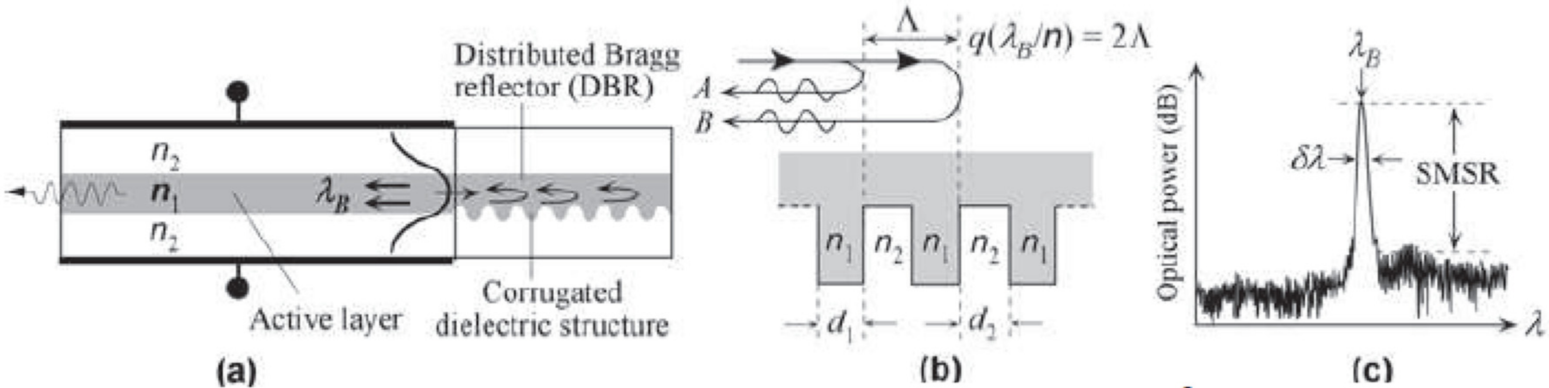
Under a sufficient forward bias, the electrons will be injected from *n*-AlGaAs, and holes from *p*-AlGaAs into the QW's CB and VB, respectively. These injected electrons very quickly thermalize and start filling states at and near E_1. Since at E_1 there is a finite and substantial density of states (g_1), **electrons in the conduction band do not have to spread far in energy to find states**. Thus, a large concentration of electrons can easily occur at E_1, whereas this is not the case in the bulk semiconductor at E_c. Similarly, the majority of holes in the valence band will be around E_1' since there are sufficient states at this energy. Thus, under a forward bias, the electron concentration at E_1 increases rapidly with the current and hence, **population inversion occurs quickly without the need for a large current** to bring in a great number of electrons.

10. Single Frequency Laser

Laser has high modal purity and hence a very narrow spectral width. Such laser diodes are often called **single frequency lasers.**

<u>distributed Bragg reflector (DBR) laser</u>

n: **effective refractive index** of the corrugated medium



<u>distributed feedback (DFB) laser</u>

$$\lambda_m = \lambda_B \pm \frac{\lambda_B^2}{2nL}(m + 1)$$

Corrugations in the refractive index of guiding layer act as **optical feedback distributed over the cavity length**.