Optical Antennas versus Microwave Antennas
——A Personal Review

Wei E.I. Sha (沙威)

College of Information Science & Electronic Engineering
Zhejiang University, Hangzhou 310027, P. R. China

On leave from EEE Department, the University of Hong Kong

Email: weisha@zju.edu.cn
Website: http://www.isee.zju.edu.cn/weisha/
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1. Function

Microwave

Microwave or radio wave antennas are electrical devices which convert electric power into radio waves, and vice versa.

Optical

Optical antennas convert freely propagating optical radiation into localized electromagnetic energy, and vice versa.

- photodetection, solar energy, light emission, sensing, microscopy, and spectroscopy

radio broadcasting, television, communication, radar, cell phone, etc
2. Basic Elements

**Microwave**

*Transmitter:* current (voltage) source

*Receiver:* electrical load

*Resonant Transducer:* half-wavelength limit

*Resonant Transducer:* scaling law

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**Optical**

*Transmitter/Receiver:* quantum dots

*Transmitter/Receiver:* atoms

*Transmitter/Receiver:* molecules

*Transmitter/Receiver:* ions

*Resonant Transducer:* break half-wavelength limit

*Resonant Transducer:* break scaling law

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Antennas satisfy reciprocity theorem.

$$\int_V E_1 \cdot J_2 dV = \int_V E_2 \cdot J_1 dV$$
3. Computational Models

**Microwave**

- far-field radiation
- perfect electric conductor
- low loss dielectric
- propagation wave interaction
- linear, single-physics, classical

**Optical**

- near-field concentration
- highly dispersive and lossy materials
- evanescent/surface wave couplings
- nonlinear, multiphysics, and quantum effects

L~λ/2

L<<λ/2
4. Directivity and Gain (1)

**Microwave**

**Directivity**

\[ D = \frac{U}{U_0} = \frac{4\pi U}{P_{\text{rad}}} \]

- \( U \): radiation intensity (W/unit solid angle)
- \( U_0 \): radiation intensity of isotropic source (W/unit solid angle)
- \( P_{\text{rad}} \): total radiated power (W)

**Gain**

\[ \text{Gain} = 4\pi \frac{U(\theta, \phi)}{P_{\text{in}}} \]

- \( U(\theta, \phi) \): radiation intensity
- \( P_{\text{in}} \): total input (accepted) power

**Radiation efficiency**

\[ \eta = \frac{P_{\text{rad}}}{P_{\text{rad}} + P_{\text{ohmic}}} \]

**Gain**

\[ \text{Gain} = \eta D \]
4. Directivity and Gain (2)

**Optical (Consistency)**

\[
D = \frac{U}{U_0} = \frac{4\pi U}{P_{rad}}
\]

Directivity

\[
\text{Gain} = \eta D
\]

Gain

---

**Optical (Difference)**

\[
\eta_{in} = \frac{P^0_{rad}}{(P^0_{rad} + P^0_l)}
\]

Internal efficiency

- \(P^0_{rad}\): power radiated by the emitter in the absence of the optical antenna
- \(P^0_l\): internal losses

\[
\eta = \frac{P_{rad}}{(P_{rad} + P_{ohmic} + P^0_l)}
\]

Radiation efficiency

optical antennas could significantly improve the radiation efficiency of a poor emitter!
5. Unidirectional Antennas: Yagi-Uda Antennas (1)

image theory and antenna array synthesis

PEC virtual surface

d = 0.5\lambda
d = 0.25\lambda

out-of-phase in-phase
5. Unidirectional Antennas: Yagi-Uda Antennas (2)

**Microwave**

R: reflector; A: driven element (feed); D: director

The distance between feed and reflector is smaller than 0.25\(\lambda\).

**Optical**

6. Broadband V.S. Wavelength Selectivity (1)

**Babinet principle** and broadband antennas

Input impedances of metal antenna and complementary aperture antenna satisfy

\[ Z_m Z_a = \frac{\eta^2}{4} \]

For self-complementary antenna, its input impedance is frequency of independent and thus the self-complementary antenna is a kind of broadband antennas.

6. Broadband V.S. Wavelength Selectivity (2)

wavelength selectivity by dielectric Yagi-Uda antenna and Fano resonance

7. Active Antennas: Electrical V.S. Optical Tunable

**Microwave**

- phased array
- plasma
- varactor diode
- liquid crystals

**Optical**

- optical force
- nonlinear effect
- liquid crystals
- graphene

References:

- Nano. Lett. 9:3914, 2009
impedance matching for *microwave antennas*

The transmitter with the transmission line is represented by an (Thevenin) equivalent generator (with \( V_G, R_G, \) and \( X_G \)).

The antenna is represented by its **input impedance** (which is frequency-dependent and is influenced by objects nearby) as seen from the generator.

\( jX_A \) represents energy stored in electric (\( E_e \)) and magnetic (\( E_m \)) near-field components; if \(|E_e| = |E_m|\) then \( X_A = 0 \) (antenna resonance).

\( R_r \) represents energy radiated into space (far-field components).

\( R_l \) represents energy lost, i.e. transformed into heat in the antenna structure.

Maximum Power Condition

\[ R_A = R_r + R_l = R_G, \quad X_A = -X_G \]

\[ P_G = P_A = \frac{V_G^2}{4R_G}, \quad P_r = P_A \frac{R_r}{(R_r + R_l)} \]
local density of states for **optical antennas**

To let quantum emitters efficiently radiate EM waves, photon local density of states (LDOS) should be enhanced. The LDOS counts the number of EM modes at the emitter point. Each EM mode can be taken as a decay channel. The more decay channels there are, the easier it is for an excited atom to emit photons via returning to its ground state.

In isotropic, inhomogeneous, and nonmagnetic medium, the LDOS is represented by the dyadic Green’s function in inhomogeneous environment

$$\rho(r_0, \omega_0) = \frac{2\omega_0}{\pi c^2} \text{Tr} \left\{\hat{m} \mathcal{G}^e(r_0, r_0; \omega_0)\right\}$$

Optical antennas could significantly boost LDOS due to the localized near-field enhancement by plasmonic effects. Blue: without antenna; Others: with antenna of different arm lengths

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9. Linear V.S. Nonlinear and Quantum Regimes (1)

1. Classical linear Maxwell equation or wave equation will be solved to model the EM response from *microwave antennas*.

2. Coupled wave equations with nonlinear sources will be solved to model the EM response from *nonlinear optical antennas*, where radiated waves and incident waves have different frequencies. The coupled wave equations for second-harmonic generation is given by

\[
\nabla^2 E^{(\omega)} + k(\omega)^2 E^{(\omega)} = -\frac{\omega^2}{\varepsilon_0 c^2} P^{(\omega),NL} - \frac{i\omega}{\varepsilon_0 c^2} J^{(\omega)}_{\text{pump}}
\]

fundamental field

\[
\nabla^2 E^{(2\omega)} + k(2\omega)^2 E^{(2\omega)} = -\frac{(2\omega)^2}{\varepsilon_0 c^2} P^{(2\omega),NL}
\]

second harmonic field

\[P^{(\omega),NL}\]

nonlinear source for downconversion process

\[P^{(2\omega),NL}\]

nonlinear source for upconversion process
9. Linear V.S. Nonlinear and Quantum Regimes (2)

**nonlinear optical antennas**: Yagi-Uda case

second harmonic radiation obeys a selection rule that the radiation is strictly zero along the incident $z$ direction if the scatterer is centrosymmetric at the $xoy$ plane.

Quantum World

• At quantum regime, when the object size is tiny small (typically smaller than 10 nm) so that “homogenized” permittivity and permeability of Maxwell equation is invalid or meaningless.

• If the field intensity is strong or the number of photons is large, semi-classical Maxwell-Schrödinger system is required to describe the light-particle interaction, where Maxwell equation is still classical.

• If the field intensity is very weak and the number of photons is quite small (vacuum fluctuation, single photon source, etc), Maxwell equation should be quantized and classical Maxwell equation breaks down.

\[
|E| \gg \frac{\sqrt{\hbar c}}{(c\Delta t)^2}
\]

strong field condition
9. Linear V.S. Nonlinear and Quantum Regimes (4)

**Semi-classical Framework — Maxwell-Schrödinger equations**

**Hamiltonian**

\[
H^s (\mathbf{A}, \mathbf{Y}, \psi, \psi^*) = H^{em} (\mathbf{A}, \mathbf{Y}) + H^q (\psi, \psi^*, \mathbf{A})
\]

\[
H^{em} (\mathbf{A}, \mathbf{Y}) = \int_v \left( \frac{1}{2\epsilon_0} |\mathbf{Y}|^2 + \frac{1}{2\mu_0} |\nabla \times \mathbf{A}|^2 \right) \, dr
\]

\[
H^q (\psi, \psi^*, \mathbf{A}) = \int_v \left[ \psi^* \left( \hat{\mathbf{p}} - q\mathbf{A} \right)^2 \frac{1}{2m} \psi + \psi^* \psi \right] \, dr
\]

**Generalized coordinate and momentum**

\[
\mathbf{q} = (\mathbf{A}, \psi_r) \quad \mathbf{p} = (\mathbf{Y}, \psi_i)
\]

\[
\frac{\partial \mathbf{p}}{\partial t} = -\frac{\partial H^s}{\partial \mathbf{q}}
\]

\[
\frac{\partial \mathbf{q}}{\partial t} = \frac{\partial H^s}{\partial \mathbf{p}}
\]

**Maxwell equation**

\[
\frac{\partial \mathbf{A}}{\partial t} = \frac{\partial H^s}{\partial \mathbf{Y}} = \frac{\mathbf{Y}}{\epsilon_0}
\]

\[
\frac{\partial \mathbf{Y}}{\partial t} = -\frac{\partial H^s}{\partial \mathbf{A}} = -\frac{\nabla \times \nabla \times \mathbf{A}}{\mu_0} + \mathbf{J}
\]

**Schrödinger equation**

\[
\frac{\partial \psi}{\partial t} = \frac{1}{i\hbar} \frac{\partial H^s}{\partial \psi^*} = \frac{1}{i\hbar} \left[ \left( \hat{\mathbf{p}} - q\mathbf{A} \right)^2 + V \right] \psi
\]

\[
\frac{\partial \psi^*}{\partial t} = -\frac{1}{i\hbar} \frac{\partial H^s}{\partial \psi} = -\frac{1}{i\hbar} \left[ \left( \hat{\mathbf{p}} + q\mathbf{A} \right)^2 + V \right] \psi^*
\]

\[
\mathbf{J} = \frac{q}{2m} \left[ \psi^* \left( \hat{\mathbf{p}} - q\mathbf{A} \right) \psi + \psi \left( -\hat{\mathbf{p}} - q\mathbf{A} \right) \psi^* \right]
\]

**Quantum current**

Quantized Maxwell equation in dispersive and lossy media by Lorentz model

Maxwell equation in time-domain

\[
\begin{align*}
\dot{\mathbf{E}} &= -\nabla \times \dot{\mathbf{H}} \\
\dot{\mathbf{H}} &= -\nabla \times \mathbf{E} \\
\dot{\mathbf{P}} &= -\omega_0 \dot{\mathbf{P}} - \eta \dot{\mathbf{P}} + \hat{F}_I + \frac{\omega_p^2}{\omega_0} \mathbf{E} \\
\dot{\mathbf{\Pi}} &= -\omega_0 \dot{\mathbf{P}} - \eta \dot{\mathbf{P}} + \hat{F}_I + \frac{\omega_p^2}{\omega_0} \mathbf{E} \\
\mathbf{F}_{\mathbf{R}} &= \omega_0 \dot{\mathbf{\Pi}} - \eta \dot{\mathbf{P}} + \hat{F}_R \\
\left\langle \left[ \hat{F}_R, \hat{F}_I \right] \right\rangle &= i2\eta \frac{\omega_p^2}{\omega_0} \delta(t-t') \mathbf{I}
\end{align*}
\]

Wave equation in frequency domain

\[
\begin{align*}
\nabla \times \nabla \times \hat{\mathbf{E}}(\mathbf{r}, \omega) - \omega^2 \epsilon(\mathbf{r}, \omega) \hat{\mathbf{E}}(\mathbf{r}, \omega) &= i\omega \hat{\mathbf{j}}_n(\mathbf{r}, \omega) \\
\hat{\mathbf{j}}_n(\mathbf{r}, \omega) &= \frac{-i\omega_0 \hat{F}_I(\mathbf{r}) - i\omega \left[ \eta(\mathbf{r}) - i\omega \right] \hat{F}_R(\mathbf{r})}{\left[ \eta(\mathbf{r}) - i\omega \right]^2 + \omega_0^2} \\
\epsilon(\mathbf{r}, \omega) &= 1 + \frac{\omega_p^2}{\left[ \eta(\mathbf{r}) - i\omega \right]^2 + \omega_0^2} \\
\end{align*}
\]

Fluctuation-dissipation theorem

\[
\left\langle \left[ \hat{\mathbf{j}}_n(\mathbf{r}, \omega), \hat{\mathbf{j}}_n^\dagger(\mathbf{r}, \omega') \right] \right\rangle = \frac{\hbar \omega}{\pi} \sigma(\mathbf{r}, \omega) \delta(\omega - \omega')
\]

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Conclusion

1. Electromagnetic theories still show great capabilities to design both microwave and optical antennas.

2. Due to dispersive and lossy materials at optical frequencies, the design of optical antennas could borrow the ideas from that of microwave antennas but needs to be optimized by rigorous full-wave simulation. The consideration of evanescent or surface wave coupling is essential to the optimized design.

3. A new principle should be explored for a new application of optical antennas, such as vibration spectra detection by wavelength selectivity through Fano resonance concept.

4. Some figures of merit of optical antennas should be modified or regenerated, such as radiation efficiency and local density of states.

5. Manipulation of nonlinear and quantum effects of optical antennas is a new emerging research area.
THANKS FOR YOUR ATTENTION!