ACKNOWLEDGMENTS

This work is supported by the Key Project of National Natural Science Fund (no. 90207004) and the National Natural Science Fund (no. 60306005), a collaborative project between Peking University and Intel Corporation. The authors thank the Institute of Microelectronics of Chinese Academy of Sciences (IMECAS) for measurement support.

REFERENCES


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A NOVEL HIGH-ORDER TIME-DOMAIN SCHEME FOR THREE-DIMENSIONAL MAXWELL’S EQUATIONS

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Received 24 November 2005

ABSTRACT: A novel high-order time-domain scheme with a four-stage optimized symplectic integrator propagator is presented for 3D electromagnetic scattering problems. The scheme is nondissipative and does not require more storage than the classical finite-difference time-domain (FDTD) method. The numerical results show the scheme has better stability and more efficiency than the classical FDTD method.

Key words: high-order time-domain scheme; symplectic integrator propagator; finite-difference time-domain (FDTD)

1. INTRODUCTION

As the most standard method, the classical finite-difference time-domain (FDTD) method, which is 2nd-order accurate both in space and time, has been thoroughly established and widely used in computational electromagnetics (CEM) up to the present [1–3]. Unfortunately, for electrically large domains and late-time analysis, the classical Yee method begins to show its limitations due to the accumulation of phase errors. One solution is to use high-order schemes. For example, in the work of Fang [4], a 4th-order central-difference approximation is proposed in conjunction with the Yee cell [1] and a modified 4th-order leapfrog integrator is employed. In deriving Fang’s method, a 3rd-order correctional temporal derivative was introduced and then converted into 3rd-order spatial derivatives through repeated application of the Maxwell’s equations. So the method is difficult to deal with perfectly matched layer (PML) absorbing boundary conditions [5] for simulating unbounded domains and in calculating the varying of the permittivity or permeability in inhomogeneous domains. Another solution is to make use of the explicit four-stage Runge–Kutta integrator to approximate the temporal derivatives together with a compact central-difference approximation with the Yee cell to the space derivatives in the Maxwell’s equations [6]. It should be noted, however, that the method produces not only phase error, but also amplitude error. Moreover, it requires additional memory for temporary storage of data for the internal stages.

Symplectic methods include a variety of different time-discretization methods designed to preserve the global symplectic structure of the phase space for a Hamiltonian system. They show substantial benefits in numerical computation for a Hamiltonian system, especially in long-term simulations. Since the Maxwell’s equations can be written as a system of an infinite-dimensional Hamiltonian system, the proper solution should be obtained using symplectic methods, which preserve the symplectic structure in the time direction. Recently, symplectic methods have been adopted for use in CEM. The advantages of symplectic methods have been verified in [7–12]. The application of symplectic integrator propagator for solving Maxwell’s Equations was made in [7] to obtain a 4th-order symplectic FDTD method, and in [13] an optimized symplectic integrator propagator coefficients was presented. In this paper, we make use of the four-stage optimized propagator coefficients in [13] to discretize 3D Maxwell’s equations in the time direction, and use a 4th-order difference operator to discretize the 1st-order space differential operators directly. The bistatic radar cross section (RCS) of a dielectric sphere are computed using the present scheme for the first time.

2. THE THEORY

Maxwell’s equations in an isotropic and sourceless medium can be written in matrix form as

\[
\frac{\partial}{\partial t} \begin{bmatrix} \mathbf{H} \\ \mathbf{E} \end{bmatrix} = \begin{bmatrix} A & B \end{bmatrix} \begin{bmatrix} \mathbf{H} \\ \mathbf{E} \end{bmatrix}, \tag{1}
\]

\[
A = \begin{bmatrix} -\mu^{-1} \sigma^e \mathbf{I}_3 & -\mu^{-1} \mathbf{R} \\ \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} \end{bmatrix}, \quad
B = \begin{bmatrix} \mathbf{0}_{3 \times 3} \\ \mathbf{0}_{3 \times 3} \end{bmatrix}, \quad
\mathbf{R} = \begin{bmatrix} e^{-R} \mathbf{I}_3 \\ e^{-R} \mathbf{I}_3 \end{bmatrix}, \tag{2}
\]

where \( \mu \) and \( \epsilon \) are the respective permeability and permittivity, \( \sigma \) and \( \sigma^e \) are the respective electric and magnetic conductivities, \( \mathbf{0}_{3 \times 3} \) is the 3 \( \times \) 3 null matrix, \( \mathbf{I}_3 \) is the 3 \( \times \) 3 unit matrix, \( \mathbf{R} \) is the 3 \( \times \) 3 matrix representing the 3D curl operator.
Since the matrices $A$ and $B$ do not commute (that is, $AB \neq BA$), in the time direction, after the temporal increment $\Delta t$, the symplectic integrator propagator can be adopted to approximate

$$
\exp(\Delta t (A + B)) = \prod_{i=1}^{n} \exp(D_i \Delta t) \exp(C_i \Delta t) + O(\Delta t^{n+1}),
$$

where $C_i$ and $D_i$ are constant coefficients of the propagator. In this paper, particularly, we use the four-stage optimized propagator coefficients in [13].

The propagators can be calculated as follows:

$$
\exp(\Delta t A) = \exp(\frac{\Delta t (\sigma\Delta t)}{\mu}) I_3 \left( \frac{1 - \exp(\frac{\Delta t (\sigma\Delta t)}{\mu})}{\sigma \Delta t} \right) R,
$$

where $\Delta t$ is the temporal increment, $\sigma$ is the conductivity, and $R$ is the spatial uniform increment. In the sourceless and lossless space, as an example, the detailed expressions of the $n$th space direction and $z$-polarized in the $x$ direction. The propagators can be calculated as follows:

$$
E_{i+1/2}^{n+1/4}(i + \frac{1}{2}, j, k) = E_{i+1/2}^{n+1/4}(i + \frac{1}{2}, j, k) + \frac{1}{\varepsilon_i} \times \left[ \text{Coef}_1 \times \left[ H_{r}^{n+1/4}(i + \frac{1}{2}, j + \frac{1}{2}, k) - H_{r}^{n+1/4}(i + \frac{1}{2}, j - \frac{1}{2}, k) \right] 
- H_{r}^{n+1/4}(i + \frac{1}{2}, j, k + \frac{1}{2}) + H_{r}^{n+1/4}(i + \frac{1}{2}, j, k - \frac{1}{2}) \right] + \text{Coef}_2 \times \left[ H_{r}^{n+1/4}(i + \frac{1}{2}, j + \frac{3}{2}, k) - H_{r}^{n+1/4}(i + \frac{1}{2}, j - \frac{3}{2}, k) \right] 
- H_{r}^{n+1/4}(i + \frac{1}{2}, j, k + \frac{3}{2}) + H_{r}^{n+1/4}(i + \frac{1}{2}, j, k - \frac{3}{2}) \right],
$$

where $n + 1/4$ is the $n$th stage calculation after the $n$th step, $\varepsilon_i$ is the local relative permittivity at point $(i + 1/2, j, k)$, and $\varepsilon = \varepsilon_{i,0}$, CFL is the Courant–Friedrichs–Levy number, and $\Delta t$ and $\Delta x$ are the temporal and uniform spatial increment, respectively. In the following numerical examples, we use a similar technique to discretize a 1D incident field based on a 4th-order approximation, and the total-field and scattered-field (TF-SF) formulation is revised according to the requirements of consistency between TF-SF boundaries. The high-order PML absorbing boundary condition is employed to solve unbounded electromagnetic scattering problems. In order to obtain the values of scattered fields on every cell’s center, high-order cubic interpolation is adopted here, and high-order near-to far-field transformation is implemented to obtain the bistatic RCS.

3. NUMERICAL RESULTS

Consider a dielectric sphere illuminated by a plane wave propagating in the $z$ direction and $E$-polarized in the $x$ direction. The frequency of the incident wave is 300 MHz. The sphere has a diameter of 1.0 m, relative permittivity $\varepsilon_\infty = 4$, and a conductivity of 0.3. The size of the cell is 5.0 cm. The total computational domain is $83 \times 83 \times 83$ cells, the total field occupies $34 \times 34 \times 34$ cells, and 10 PML layers are implemented. A Mic series is presented as an analytical solution. Figure 1 shows the bistatic RCS in the E-plane simulated within 1700 time steps. The results computed by our scheme and FDTD are in very good agreement with the analytical solution when CFL is 0.5. The global relative root-mean-squares RCS error is 0.2123 using our scheme, as compared to 0.2247 using the classical FDTD method. In Figure 2, the results calculated by the FDTD method fail to be consistent with the analytical solution where CFL = 0.8 and 1.0 ($>1/\sqrt{3}$), the limit of CFL is classical FDTD method), while our scheme still complies with the analytical solution. Clearly, the results demonstrate that the present scheme has better stability compared with

![Figure 1 E-plane bistatic RCS of the dielectric sphere with CFL = 0.5](image-url)
the classical FDTD method and the error achieved by our scheme is smaller than that by the classical FDTD method.

4. CONCLUSION

A novel high-order time-domain scheme has been presented. The scheme was obtained by discretization of Maxwell’s equations with a four-stage optimized symplectic integrator propagator in the time direction, and a 4-th-order difference operator in order to discretize the 1-st order space differential operators directly. The scheme is nondissipative and does not require more storage than the classical FDTD method. Especially, the scheme has better stability and more efficiency than the classical FDTD method.

The major shortcoming of the scheme is that it consumes more CPU time than the classical FDTD method when the same cell size is used. An effective parallel algorithm is an open question for further study.

ACKNOWLEDGMENTS

This work was partially supported by the National Natural Science Foundation of China (No. 60371041).

REFERENCES