# ACCURATE COMPUTATION OF WIDE-BAND RESPONSE OF ELECTROMGANETIC SCATTERING PROBLEMS VIA MAEHLY APPROXIMATION

**Mingsheng Chen, Xianliang Wu, Zhixiang Huang, and Wei Sha** Key Laboratory of Intelligent Computing and Signal Processing, Anhui University, Hefei, 230039, People's Republic of China

Received 9 October 2006

ABSTRACT: The best uniform rational approximation, performed by the Maehly approximation, is applied to the method of moments (MOM) to obtain the radar cross section (RCS) over a broad frequency band. Numerical results for three dimensional arbitrarily shaped perfectly electrically conducting (PEC) bodies are considered. Compared with the asymptotic waveform evaluation (AWE) technique, the presented technique is found to be efficient in much broader frequency band with lower memory required. © 2007 Wiley Periodicals, Inc. Microwave Opt Technol Lett 49: 1144–1146, 2007; Published online in Wiley Inter-Science (www.interscience.wiley.com). DOI 10.1002/mop.22367

**Key words:** *Maehly approximation; method of moments; asymptotic waveform evaluation; wide-band radar cross section* 

#### 1. INTRODUCTION

In many practical applications, it is desirable to determine the scattering from an object over a wide frequency band. The solution of surface integral equation (SIE) via the method of moments (MOM) has been a very useful tool to complete the scattering analysis of arbitrary shaped objects [1, 2]. To obtain the radar cross section (RCS) over a broad frequency band using MOM, one has to repeat the calculations at each frequency point, which must be computational intensive. For years there is a need to find approximate solution techniques that can efficiently simulate a frequency response over a broad band.

Among the earlier attempts to achieve a fast frequency sweep analysis by MOM and other frequency-domain techniques, the impedance matrix interpolation technique and the asymptotic waveform evaluation (AWE) technique are prevalent. The impedance matrix interpolation technique is still time-consuming since the solution of the algebraic equations at each frequency point. In the AWE technique, the electric current is expanded in the Taylor series around a frequency, and the rational function approach is used to improve the accuracy. As compared with using MOM at each of the frequency points, the AWE method is found to be superior in terms of the CPU time to obtain frequency response [3–6]. However, the accuracy of the Taylor series is limited by the radius of convergence, and the high derivatives of the dense impedance matrix must be storied, which will greatly increase the memory needed.

In this letter, a fast frequency-sweep technique was presented to alleviate this problem. This technique applies the Maehly approximation technique to the MOM solution of the SIE for different PEC objects. For a given frequency band, the frequency points corresponding to the Chebyshev nodes are found by transformation of coordinates, and MOM is used to compute the electric currents at these points, then the surface current is represented by the Chebyshev series, which can be as good as the best polynomial approximation. Akin to the padé approximation, the coefficients of the Chebyshev series are then matched via the Maehly approximation to a rational function to improve the accuracy, which can be used as the best rational approximation [7, 8]. Using the rational function, the electric current distribution can be obtained at any frequency within the bandwidth. Using this current distribution, the RCS is obtained. Numerical results for a sphere and two cubes are considered.

### 2. FORMULATION

Consider the problem of electromagnetic scattering by an arbitrarily shaped three-dimensional PEC body illuminated by an incident field  $\mathbf{E}^{\text{inc}}$ . The electric field integral equation (EFIE) is given by

$$\mathbf{\hat{n}} \times L(\mathbf{J}) = \mathbf{\hat{n}} \times \mathbf{E}^{\text{inc}}(\mathbf{r}) \tag{1}$$

where  $\mathbf{J}(\mathbf{r})$  denotes the unknown surface current density and the integral operator *L* is defined by

$$L(\mathbf{J}) = jk_0\eta_0 \iiint_{S} \left( \mathbf{J}(\mathbf{r}')g(\mathbf{r},\mathbf{r}') + \frac{1}{k^2} \nabla' \cdot \mathbf{J}(\mathbf{r}') \nabla g(\mathbf{r},\mathbf{r}') \right) dS' \quad (2)$$

in which *k* is the free-space wavenumber,  $\eta_0$  is the free-space wave impedance,  $\hat{\mathbf{n}}$  is an outwardly directed normal,  $g(\mathbf{r}, \mathbf{r}')$  is the well-known free-space Green's function.

The surface current density can then be expanded using the Rao–Wilton–Glisson (RWG) basis functions [9]. Applying Galerkin's method to Eq. (1) results in

$$\mathbf{Z}(k)\mathbf{I}(k) = \mathbf{V}(k) \tag{3}$$

where  $\mathbf{Z}(k)$  is the impedance matrix,  $\mathbf{V}(k)$  is the excitation column vector.

For a given frequency band  $f \epsilon [f_a, f_b]$ ,  $k \epsilon [k_a, k_b]$ , the transformation of coordinates is applied as

$$\tilde{k} = \frac{2k - (k_a + k_b)}{k_b - k_a} \tag{4}$$

such that  $\tilde{k} \in [-1,1]$ .

Then the Chebyshev approximation for I(k) is given by

$$\mathbf{I}(k) = \mathbf{I}\left(\frac{\tilde{k}(k_b - k_a) + (k_b + k_a)}{2}\right)$$
$$\approx \sum_{l=0}^{n} c_l T_l(\tilde{k}) - \frac{c_0}{2}$$
(5)

$$c_{l} = \frac{2}{n+1} \sum_{i=0}^{n+1} \mathbf{I}(k_{i}) T_{l}(\tilde{k}_{i})$$
(6)

$$T_0(x) = 1, T_1(x) = x, \cdots, T_{n+1}(x) = 2xT_n(x) - T_{n-1}(x)$$
 (7)

where  $\tilde{k}_i (i = 0, 1, 2, \dots, n)$  are the Chebyshev nodes for  $T_n(\tilde{k})$ , and  $k_i \in [k_a, k_b]$  can be obtained by substituting  $\tilde{k}_i$  into (4).

To improve the accuracy of the numerical solution, the Chebyshev series is replaced by a rational function which is called Maehly approximation

$$\mathbf{I}(k) \approx R_{LM}(\tilde{k}) = \frac{P_L(k)}{Q_M(\tilde{k})} = \frac{a_0 T_0(\tilde{k}) + a_1 T_1(\tilde{k}) + \dots + a_L T_L(\tilde{k})}{b_0 T_0(\tilde{k}) + b_1 T_1(\tilde{k}) + \dots + b_M T_M(\tilde{k})}$$
(8)

where  $b_0$  is set to be 1 as the rational function can be divided by an arbitrary constant.

Substituting Eq. (8) into (5) and using the identity

$$T_p(x)T_q(x) = \frac{1}{2}(T_{p+q}(x) + T_{p+q}(x))$$
(9)

The unknown coefficients  $a_i(i = 0, 1, \dots, L)$  and  $b_j(j = 1, 2, \dots, M)$  can be solved by

$$\begin{cases} a_0 = \frac{1}{2}b_0c_0 + \frac{1}{2}\sum_{p=1}^M b_pc_p \\ a_q = c_q + \frac{1}{4}b_qc_0 + \frac{1}{2}\sum_{p=1}^M b_p(c_{p+q} + c_{1p-q}) \quad q = 1, 2, \cdots, L \end{cases}$$
(10)

$$\frac{1}{2}\sum_{p=1}^{M} (c_{L+p+q} + c_{L+q-p})b_p + c_{L+q} = 0 \quad q = 1, 2, \cdots, M \quad (11)$$

The error function is given by

$$E_n = I(k) - R_{LM}(\tilde{k}) = \frac{1}{Q_M(\tilde{k})} \sum_{p=L+M+1}^{\infty} h_p T_p(\tilde{k})$$
(12)

where the coefficient  $h_p$  is defined by

$$P_M(\tilde{k}) - Q_L(\tilde{k})I(k) = \sum_{p=L+M+1}^{\infty} h_p T_p(\tilde{k})$$
(13)

Since  $b_0 = 1$ , and  $h_p$  attenuates quickly, the error function can be approximated by

$$E_n \approx h_{L+M+1} T_{L+M+1}(\tilde{k}) \tag{14}$$

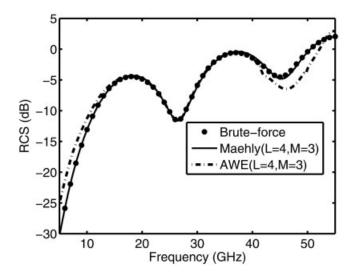


Figure 1 RCS frequency response of the sphere with radius of 0.318 cm

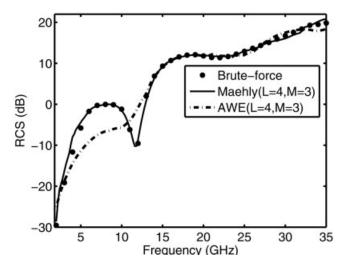


Figure 2 RCS frequency response of two cubes

$$h_{L+M+1} = c_{L+M+1} + \frac{1}{2} \sum_{i=1}^{M} b_i (c_{L+M+i+1} + c_{L+M-i+1})$$
(15)

Hence  $R_{LM}(\bar{k})$  can be used as the best uniform rational approximation of I(k) [8].

## 3. NUMERICAL EXAMPLES

The first example is a PEC sphere of radius 0.318 cm, which is illuminated by a plane wave propagating in the *z* direction and E-polarized in the *x* direction. The sphere is discretized into 500 triangular elements which result in 750 basis functions.

As shown in Figure 1, with a frequency step of 1 GHz, it takes the brute-force method (in which, the solution to the matrix equation is obtained at each of the frequency point) 380.84 sec to obtain the solution from 5 to 55 GHz. With one expansion point at 18.5 GHz and a seventh-order Taylor expansion (padé (4, 3)), the AWE gives an efficient solution with 0.1 GHz increments from 15 to 40 GHz in 57.16 sec, while Maehly approximation (L = 4, M = 3) produces an accurate solution with 0.1 GHz increments over the entire band in 64.52 sec.

As the second example, a target that is made by two cubes is considered. The two cubes are uniformly placed on y-axis, the length of the cube is chosen to be 1 cm, and the center distance of them is 5 cm. The target is illuminated by a plane wave propagating in the z direction and E-polarized in the x direction. Under this condition, the accuracy of AWE will severely limited by the radius of convergence in Taylor series.

As shown in Figure 2, the AWE (L = 4, M = 3) obtains accurate results only in a narrow frequency band (13–30 GHz), while the Maehly method (L = 4, M = 3) produces an accurate solution from 2 to 35 GHz. The target is discretized into 966 triangular elements. The CPU time required for brute-force method is 1310.46 sec, and the CPU times for AWE and Maehly are 229.63 and 241.04 sec, respectively.

All the computations reported above were achieved by VC++ 6.0 on a PIV2.66G personal computer.

#### 4. CONCLUSION

A new method is presented to compute the frequency response using a frequency-domain method such as MOM. Compared with the traditional method, the presented technique can obtain accurate results within much broader frequency band without increasing any memory, and it was shown that the use of Maehly method can speed up the calculations as much as AWE.

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# LTCC BANDPASS FILTER USING 3D COUPLED HELICAL INDUCTORS

SeungChul Choi,<sup>1</sup> JungEun Youm,<sup>1</sup> Keun Heo,<sup>1</sup> JuHwan Lim,<sup>1</sup> ChulSoon Park,<sup>2</sup> and SungWoo Hwang<sup>1</sup>

- <sup>1</sup> School of Electrical Engineering, Korea University, 5–1 Anam, Sungbuk, Seoul 136–701, Korea
- <sup>2</sup> School of Engineering, Information and Communications University, 119 Munji-ro, Yuseong, Daejon 305–732, Korea

Received 9 October 2006

**ABSTRACT:** A new low-temperature co-fired ceramic (LTCC) bandpass filter using three-dimensionally coupled helical inductors is presented. The resonator is a via-connected multilayer helical inductor and the filter uses mixed coupling between face-to-face inductors. The fabricated filter exhibits the center frequency of 5.8 GHz, 3-dB bandwidth of 800 MHz, and the insertion loss of 1.9 dB. The size of the active region is  $1.7 \times 2.3 \times 0.8$  mm<sup>3</sup>. The measured results are consistent with the three-dimensional electromagnetic simulation and the equivalent circuit modeling. © 2007 Wiley Periodicals, Inc. Microwave Opt Technol Lett 49: 1146–1147, 2007; Published online in Wiley InterScience (www. interscience.wiley.com). DOI 10.1002/mop.22366

Key words: low-temperature co-fired ceramic (LTCC); helical inductor

## **1. INTRODUCTION**

Recent trend in wireless communication system components is compact integration of various functions in a single package or module. Low temperature co-fired ceramic (LTCC) is highly suitable for such system-in-package (SiP) application, since it is possible to embed passive components in various types of multilayered packages. Bandpass filter (BPF) is a most important passive component in wireless front-end circuits. There has been huge effort in the development of BPFs for performance improvement and for miniaturization. Most of passive RF filters utilize either combinations of discrete components or coupling of certain resonators fabricated on planar substrates [1, 2]. On the other hand, unique stacking capability of LTCC technology gives us an opportunity to test the possibility of three-dimensional (3D) coupling. In this paper, we demonstrate that 3D coupled helical inductor BPF can be realized by LTCC technology.

## 2. FILTER DESIGN AND FABRICATION

Figure 1 shows a 3D schematic of the proposed BPF. Two parallel sets of face-to-face helical inductors (total 4 inductors) are shown. The helical inductor is realized by LTCC vias [3]. At high frequencies, these inductors act as resonators, the resonant frequency of which is determined by the inductance and the parasitic capacitances of the inductor. There is 3D coupling between the face-to-face inductors. The efficiency of the coupling is a function of the mutual inductance and the coupling capacitance between resonators, which are also functions of the distance between them [2].

The design of the BPF is performed by utilizing 3D electromagnetic (EM) simulation of the filter structures with various dimensions. The starting point of this EM optimization is a rough estimation of the inductance based on our LTCC library, and a usual shrinkage factor of the process is considered. A seven-layer LTCC technology is used for the fabrication. The

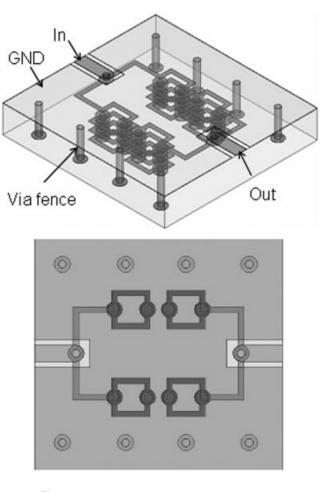


Figure 1 Schematic of the proposed bandpass filter