APPLICATION OF THE HIGH-ORDER SYMPLECTIC FDTD SCHEME TO THE CURVED THREE-DIMENSIONAL PERFECTLY CONDUCTING OBJECTS

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ABSTRACT: A high-order symplectic finite-difference time-domain (SFDTD) scheme using the diagonal split-cell model is presented to analyze electromagnetic scattering of the curved three-dimensional perfectly conducting objects. On the one hand, for the undistorted cells, the fourth-order accurate spatial difference is employed. On the other hand, for the completely distorted cells, the treatment of the curved surfaces is based on the diagonal split-cell model. Finally, for the partially distorted cells, the interpolation strategy is proposed to keep the field components continuous. The numerical experiments suggest that the diagonal SFDTD scheme can obtain more accurate results than both the staircase SFDTD scheme and the traditional diagonal FDTD method. Furthermore, in view of the high numerical stability, the improved symplectic scheme does not need to decrease time increment to comply with the stability criterion.

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Key words: symplectic integrator; curved conducting surfaces; diagonal split-cell model; high-order difference; radar cross section

1. INTRODUCTION

As the most standard algorithm, the traditional finite-difference time-domain (FDTD) method [1], which is second-order accurate in both space and time, has been widely applied to the electromagnetic computation and simulation. Unfortunately, for electrically large domains and for long-term simulation, the method consumes large computational resources owing to the limit of numerical dispersion and stability. Up to now, some more efficient solutions have been presented. For example, in 1989, Fang proposed high-order accurate FDTD method [2], which is fourth-order accurate in both space and time. But the method is hard to treat the varying of permittivity and permeability in the inhomogeneous domain on account of the application of third-order spatial derivatives to substitute for third-order correctional temporal derivatives. Another approach is to use fourth-order accurate Runge-Kutta method [3] in the time direction and central-difference with Yee lattice in the space direction; yet, the method is dissipative and requires additional memory.

The symplectic schemes have demonstrated their advantage in energy conservation for the Hamiltonian system over other high-order methods [4]. A symplectic FDTD (SFDTD) scheme [5, 6], which is explicit fourth-order accurate in both space and time, was introduced to the computational electromagnetism by Hiroto for analyzing waveguide’s eigenmode. It has been verified that the SFDTD scheme is nondissipative and saves memory. Moreover, the total field and scattered field technique and the near-to-far-field transformation [7, 8] have been further developed, by which the radar cross section (RCS) of dielectric sphere was successfully computed.

However, considering the high-order difference approximation for the spatial derivatives and the staircase model for the curved surfaces, the advantage of the SFDTD scheme cannot extend to electromagnetic scattering of the curved three-dimensional perfectly conducting objects. Here, the diagonal SFDTD scheme is presented to overcome the problem. The Yee cells in the scheme are classified and discriminatingly handled, which not only eliminates the spurious solutions, but also maintains the numerical results accurate.

The paper is organized as follows. The formulation of the SFDTD scheme is given in section 2, followed by the diagonal split-cell model for treating curved surfaces specified in section 3, numerical results are presented in section 4, and summary is concluded in section 5.

2. GENERAL FORMULATION

The following notation for a function of space and time on a Cartesian grid is introduced as

\[ F(x,y,z,t) = F(i\Delta_x,j\Delta_y,k\Delta_z,(n + l/m)\Delta_t) \]  

where \( \Delta_x, \Delta_y, \Delta_z \) are, respectively, the lattice space increments in the \( x, y, \) and \( z \) coordinate directions, \( \Delta_t \) is the time increment, \( i, j, k, n, l, m \) are integers, \( n + l/m \) denotes the \( l \rightarrow th \) stage after the \( n \rightarrow th \) time step, and \( m \) is the total stage number.

As for the space direction, centered finite-difference expressions are used to discretize the first-order spatial derivatives, as follows

\[ \frac{\partial F^{n+\frac{1}{2}m}}{\partial \delta} = \lambda_1 \frac{F^{n+\frac{1}{2}m}(h + 1/2) - F^{n-\frac{1}{2}m}(h - 1/2)}{\Delta_t} + \lambda_2 \frac{F^{n+\frac{1}{2}m}(h + 3/2) - F^{n-\frac{1}{2}m}(h - 3/2)}{3\Delta_t} \]

where \( \delta = x,y,z \), \( h = i,j,k \), and \( \lambda_1 + \lambda_2 = 1 \). In addition, when \( \lambda_1 = 1 \), the expressions are second-order accurate in space, and when \( \lambda_1 = 9/8 \), those are fourth-order accurate in space.

Maxwell’s equations in the free space can be written in matrix form as [6]

\[ \frac{\partial}{\partial t} \begin{pmatrix} H \\ E \end{pmatrix} = (A + B) \begin{pmatrix} H \\ E \end{pmatrix} \]

\[ A = \begin{pmatrix} [0]_{3 \times 3} \\ R \end{pmatrix}, \quad B = \begin{pmatrix} [0]_{3 \times 3} \\ [0]_{3 \times 3} - \mu_0 R \end{pmatrix} \]

\[ R = \begin{pmatrix} 0 & -\frac{\partial}{\partial z} & \frac{\partial}{\partial y} \\ 0 & \frac{\partial}{\partial z} & -\frac{\partial}{\partial x} \\ -\frac{\partial}{\partial y} & \frac{\partial}{\partial x} & 0 \end{pmatrix} \]

where \( E = (E_x, E_y, E_z)^T \) is the electric field vector and \( H = (H_x, H_y, H_z)^T \) is the magnetic filed vector, \( \varepsilon_0 \) and \( \mu_0 \) are the permittivity and permeability of the free space, \( [0]_{3 \times 3} \) is the 3 \( \times \) 3 null matrix, and \( R \) is the 3 \( \times \) 3 matrix representing the three-dimensional curl operator.
Using the product of elementary symplectic mapping, the exact solution of Eq. (3) during the time step $\Delta t$ can be approximately constructed as [9]

$$\exp(\Delta t(A + B)) = \prod_{i=1}^{m} \exp(c_i \Delta t B) \exp(d_i \Delta t A) + O(\Delta t)^{p+1}$$

where $c_i$ and $d_i$ are the constant coefficients of the symplectic integrator, $p$ is the order of the approximation. Here we use $m = 5$ and $p = 4$, a five-stage fourth-order symplectic integrator is obtained. The coefficients can be found in [6].

The interpolation of the $x$ component of the normalized electric field ($\vec{E}_x = \frac{\vec{E}_x}{\sqrt{\varepsilon_0/\mu_0 E_0}}$) for the SFDTD scheme can be written as

$$E_{x,i}^{n+1/m} = E_{x,i}^{n+(l-1)/m}(i + \frac{1}{2}, j, k) + d_j \times \left( \lambda_i \times \left[ \text{CFL}_y \right. \right.$$

$$\times \left( H_{ri}^{n+(l-1)/m}(i + \frac{1}{2}, j, k) - H_{ri}^{n+(l-1)/m}(i + \frac{1}{2}, j - \frac{1}{2}) \right)$$

$$- \text{CFL}_y \times \left( H_{ri}^{n+(l-1)/m}(i + \frac{1}{2}, j + \frac{1}{2}) - H_{ri}^{n+(l-1)/m}(i + \frac{1}{2}, j - \frac{1}{2}) \right)$$

$$- \frac{1}{2} \left( \lambda_i \right) \times \left( \text{CFL}_x \times \left( \frac{H_{ri}^{n+(l-1)/m}(i + \frac{1}{2}, j, k) - H_{ri}^{n+(l-1)/m}(i + \frac{1}{2}, j - \frac{1}{2})}{2} \right)$$

$$- \frac{3}{2} \left( \lambda_i \right) \times \left( \text{CFL}_z \times \left( \frac{H_{ri}^{n+(l-1)/m}(i + \frac{1}{2}, j, k) - H_{ri}^{n+(l-1)/m}(i + \frac{1}{2}, j - \frac{1}{2})}{2} \right) \right) \right)$$

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scheme is 0.0556 compared with 0.0816 for the traditional diagonal FDTD method.

2. The monostatic RCS of conducting sphere with the radius of 1 m is calculated. The space increment is unchanged and the CFL number is retaken to CFL$_s$ = 0.70. The Figure 4 shows the location of spatial cells for the $H_z$ component. From the Figure 5, because of the utilization of low-order spatial difference near the curved boundaries and the symplectic structure in the time direction, the SFDTD solution at the frequency higher than 300 MHz still keeps accurate and stable after 5000 time steps.

3. The proposed scheme is employed to analyze the far response of conducting prism displayed in Figure 6. The side length of the prism are, respectively, 1 m, 1 m, and $\frac{\sqrt{2}}{2}$ m, and the height is chosen to be 1 m. Under the same relative error condition, the diagonal SFDTD scheme occupies 71 $\times$ 71 $\times$ 71 cells with $\Delta_x = 1.0/8.0$ and CFL$_s$ = 0.65, by contrast, the traditional staircased FDTD approach occupies 111 $\times$ 111 $\times$ 111 cells with $\Delta_x = 1.0/15.1$ and CFL$_s$ = 0.50. About 32.4% memory and 39.5% CPU time are saved by the proposed scheme.

5. CONCLUSION

The high-order SFDTD scheme using the diagonal split-cell model can accurately and efficiently solve the scattering of three-dimensional perfectly conducting objects with curved metal boundaries. The high numerical stability of the scheme can obviate the instability problem due to the traditional diagonal approximation. Furthermore, the improved SFDTD scheme is easy to implement and places little additional computations on the original scheme. The
future work will focus on the development of the proposed scheme in conjunction with the subgridding method.

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A HAIRPIN LINE WIDEBAND BANDPASS FILTER DESIGN WITH EMBEDDED OPEN STUBS

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ABSTRACT: In this paper, a compact three-poles hairpin line wideband bandpass filter with several embedded open stubs to improve the stopband is designed and implemented on print circuit board substrate. This filter at center frequency f0 of 4.25 GHz has presented almost very good measured characteristics, including the bandwidth of 3.1–5.4 GHz (3-dB fractional bandwidth of 54%), low insertion loss of −0.7 ± 0.4 dB, sharp rejection due to two transmission zeros in the passband edge created by interstage coupling, and wide stopband rejection greater than 15 dB from 5.6 to 11 GHz. Experimental results of the fabricated filter show a good agreement with the predict results. © 2007 Wiley Periodicals, Inc. Microwave Opt Technol Lett 49: 934–936, 2007; Published online in Wiley InterScience (www.interscience.wiley.com). DOI 10.1002/mop.22302

Key words: hairpin line; stopband; bandwidth; passband

1. INTRODUCTION
New ultrawideband radar and high data-rate communication systems require very specialized RF circuits capable of operating over wide frequency ranges [1, 2]. The design of wide-band microwave filters has been a huge challenge over the last few years. Thus, several wideband filter realizations exhibiting an extremely large passband-width have been proposed using different technologies, such as microstrip or uniplanar strip [3, 4]. However, these filters in fact have many problematic issues, such as unexpected slow passband attenuation rate, spurious response, and so on. These filters may use additional bandstop filter or defect ground structure (DGS) to improve out-of-band performance, but also cause the disadvantages, such as additional device area and package problems [5].

In this letter, we report a hairpin line wideband bandpass filter (HL-WBF) with embedded open stubs (OSs), which achieves the advantages of compact size, wide passband, high passband selectivity, and wide out of band (stopband), simultaneously. After optimization the filter design, the designed filter is executed to experimentally verify the simulated results of our design.

2. ANALYSIS OF HL-WBF FILTER
Figure 1 depicts the schematic of the proposed three-poles HL-WBF with embedded OSs. In this paper, a RT/Duroid 5880 substrate with a relative dielectric constant of 2.2, a loss tangent of 0.001, and a thickness of 0.787 mm, is used for the simulation and central position of folding resonator. Therefore, the fundamental attenuation and several spurious modes of OS can be
created as attenuation poles [7]. In another words, by changing the physical length of the OS, one can control the attenuation poles in the passband edge or stopband of the HL-WBF. Therefore, two modulated parameters, spacing (s) and lengths of the OSs (d₁ and d₂), are used to modulate and achieve the coupling coefficient and stopband in this study, respectively.

A complete full-wave electromagnetic simulator [8] is used for accurate coupling coefficient as functions of the resonator spacing (s). Figure 2 shows the simulated coupling coefficient of two folding resonators. The coupling between folding resonators can be specified by the two dominant resonant modes, which are split off from the resonance condition. The simulated coupling coefficient \( K_{i,i+1} \) between the resonator \( i \) and \( i + 1 \) (\( i = 1 \) and 2) as functions of spacing (s) is calculated as [6]

\[
K_{i,i+1} = \left( \frac{f_{H}^i - f_{L}^i}{f_{H}^i + f_{L}^i} \right)
\]  

(1)

where the \( f_{H}^i \) is the higher frequency of the two resonant modes, and \( f_{L}^i \) is the lower one. It is note that the \( K_{i,i+1} \) is applied by separating two resonators at a time and that \( K_{12} \) and \( K_{23} \) are also subject to an offset s. As reducing the s value, the coupling coefficient would be increased, namely, the coupling energy can be enhanced.

A three pole HL-WBF is designed to have a fractional bandwidth of 50% at a central frequency \( f_{0} \) = 4 GHz. A three-pole \( (n = 3) \) Chebyshev lowpass prototype with a passband ripple of 3 dB is chosen. The lowpass prototype parameters are \( g_{0} = g_{4} = 1 \), \( g_{1} = g_{3} = 3.487 \), \( g_{2} = 0.7117 \). Having obtained the lowpass parameters, theoretical coupling coefficient \( M_{i,i+1} \) is obtained as \( M_{1,2} = M_{2,3} = 0.3 \), calculated by [6]

\[
M_{i,i+1} = \frac{\text{FBW}}{\sqrt{\text{FBW}} + 1}
\]  

(2)

Therefore, the spacings between the resonator \( i \) and \( i + 1 \) are also selected as \( s_{1,2} = s_{2,3} = 0.1 \) mm from Figure 2. To achieve the central frequency \( f_{0} = 4 \) GHz, the length of the half-guided-wavelength \( (\lambda_{g}/2) \) resonator is 28.8 mm. The two enhancing coupling input/output ports are designed for 50 \( \Omega \) and the gap of the two enhancing coupling input/output ports is designed for more enhancing the coupling level of the passband skirt by using a complete full-wave electromagnetic simulation. It is noted that such small gap (\( g = 0.1 \) mm) between the input/output ports and the resonators is still available by using the conventional carving machine, without using expensive lithography process [3]. Figure 3 shows simulated frequency response of HL-WBF filter without and with OSs. As expected, the slow attenuation rates of passband edge and the spurious response are appeared since the length of resonators in HL-WBF without OS is around the half guided wavelength. Therefore, creation of attenuation poles near the passband edge and harmonics are needed.

In this study, OSs added to the HL-WBF filter are used to produce attenuation poles in the high passband edge and harmonic frequency response; therefore, the proposed HL-WBF filter would have wide stopband and very high passband selectivity performances. Since the OS is close to a quasi-quarter-wavelength \( (\lambda_{g}/4) \) resonator, the attenuation pole at 5.6 GHz in the higher passband edge is obtained when the length of one OS (\( d_{1} \)) is 9.9 mm. Thus, the HL-WBF has very high passband selectivity. But, the stopband is not large enough because of insufficient attenuation ability. As the \( d_{1} \) of OS1 is 9.9 mm and \( d_{2} \) of OS2 is 5.4 mm, a broad stopband of 5.5–11 GHz is obtained.

3. EXPERIMENTAL RESULTS AND DISCUSSION

Using the above structural parameters, the designed filter was fabricated, as discussed in Figure 4, and then measured by an HP8510C Network Analyzer. Figure 4 shows the picture of the fabricated HL-WBF filter. The whole size of the fabricated HL-WBF filter with OSs is 9.3 mm × 13.2 mm, i.e., approximately...
0.17 \lambda_g by 0.24 \lambda_g, where \lambda_g is the guided wavelength at the center frequency.

Figure 5 shows the simulated and measured results of the fabricated filter. The measured results show a center frequency $f_0$ of 4.25 GHz, very low insertion loss of $-0.7 \pm 0.4$ dB, wide bandwidth of 3.1–5.4 GHz (3-dB fractional bandwidth = 54%), and wide stopband rejection greater than 15 dB from 5.6 to 11 GHz. Moreover, the attenuation poles is clearly observed in the higher side of passband edge at 5.7 GHz with $-27$ dB attenuation, indicating a selectivity of 100 dB/GHz attenuation slopes in higher passband edge. The measured results verify the possibility of the proposed designed concept, indicating the proposed filter has a good potential for broadband communications, and can be realized on the printed circuit board substrate without using expensive lithography process.

4. CONCLUSIONS

In this paper, a three-poles HL-WBF with embedded OSs having high selectivity and wide stopband is reported. By using the OSs, it is able to place attenuation poles near the passband edge so that high passband selectivity with fewer resonators could be obtained. This designed filter was fabricated and measured, showing good characteristics including the bandwidth of 3.1–5.4 GHz (3-dB fractional bandwidth = 54%), low insertion loss of $-0.7 \pm 0.4$, sharp rejection, and wide stopband rejection greater than 15 dB from 5.6–11 GHz.

Figure 5  Simulated and measured frequency response of the designed HL-WBF filter. The physical size parameters are as follows: $d_1$ of OS1 = 9.9 mm and $d_2$ of OS2 = 5.4 mm. Others parameters are shown in Figure 1

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