Asymptotic Waveform Evaluation Technique Based on Fast Lifting Wavelet Transform

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Abstract: The asymptotic waveform evaluation (AWE) technique based on fast lifting wavelet transform (LWT) is applied to the method of moments to solve the combined-field integral equation (CFIE). The wide-band radar cross section of an arbitrarily shaped two-dimensional perfectly electric conduct object is calculated. The employment of CFIE eliminates the interior resonance problems. Numerical results presented in this paper are compared with the results obtained by the method of moments. It is shown that the computational efficiency is improved greatly.

I. INTRODUCTION

The solution of the combined-field integral equation (CFIE) via the method of moments (MOM) has been a very useful method for accurately predicting the radar cross section (RCS) at a certain frequency [1], but many electro-magnetic applications require the computation of frequency responses over a broad band rather than at one or a few isolated frequencies. To obtain the RCS over a wide band using MOM, a set of algebraic equations must be solved repeatedly, which will greatly increase the central processing unit time and the storage required. Therefore, there is a need to find approximate solution techniques that can efficiently simulate a frequency response over a broad band.

Over past few years, a similar technique called asymptotic waveform evaluation (AWE) has been proposed for the timing analysis of very large scale integration (VLSI) circuits [2], [3]. Recently, a detailed description of AWE applied to frequency-domain electromagnetic analysis was presented in [4], [5]. The traditional AWE presents great superiority when the electrical size of the problem is small enough. But the interior resonance problems take place frequently as the target size increasing, and the dimension of the dense impedance matrix also increased. Based on these facts, the traditional AWE almost can hardly deal with electrically large targets. Therefore, a new method called asymptotic waveform evaluation technique based on fast lifting wavelet transform (LWT-AWE) is presented here [6], in which the combined-field integral equation (CFIE) is reduced to a matrix equation, and the LWT is applied to the equation to get a new sparse matrix equation. Then the AWE technique is applied to the new equation, and finally, the inverse LWT is employed to obtain the electric current distribution quickly at any frequency point within the given frequency band. Numerical results are compared with the results obtained by the method of moments; CPU time and storage required are decreased drastically.

II. FORMULATION

For a perfectly conducting object, the CFIE can be shown to be

$$-\hat{\mathbf{n}}\times\mathbf{H}^{S}(\mathbf{J})-\frac{\alpha}{\eta}\mathbf{E}^{S}_{\tan}(\mathbf{J}) = \hat{\mathbf{n}}\times\mathbf{H}^{i}+\frac{\alpha}{\eta}\mathbf{E}^{i}{}_{\tan} \quad (1)$$

where $\hat{\mathbf{n}}$ is the surface normal, $\mathbf{E}^i$ and $\mathbf{H}^i$, denote the incident electric and magnetic fields respectively, and η is the wave impedance. The weighting parameter α can be viewed as an arbitrary real constant range between 0 and 1 [7].

AWE implementation

By MOM method, Eq.1 can result in a matrix equation in the following form:

$$\mathbf{Z}(k)\mathbf{I}(k)=\mathbf{V}(k)\,. \quad (2)$$

Now let us consider a wavelet matrix transform; Eq.2 is transformed to

$$\tilde{\mathbf{Z}}(k)\tilde{\mathbf{I}}(k)=\tilde{\mathbf{V}}(k) \quad (3)$$

where $\tilde{\mathbf{Z}}(k)=\mathbf{W}\,\mathbf{Z}(k)\,\mathbf{W}^H$, $\tilde{\mathbf{I}}(k)=\mathbf{W}\,\mathbf{I}(k)$, and $\tilde{\mathbf{V}}(k)=\mathbf{W}\mathbf{V}(k)$; $\mathbf{W}$ is assumed to be a $N\times N$ orthogonal wavelet matrix.

The ith derivative of $\tilde{\mathbf{Z}}$ and the nth derivative of $\tilde{\mathbf{V}}$ can be computed by

$$\tilde{\mathbf{Z}}^{(i)}(k)=\mathbf{W}\,\mathbf{Z}^{(i)}(k)\,\mathbf{W}^H\,, \quad (4)$$

$$\tilde{\mathbf{V}}^{(n)}(k)=\mathbf{W}\,\mathbf{V}^{(n)}(k)\,. \quad (5)$$

To obtain the solution of (3) over a wide frequency band, we expand $\mathbf{I}(k)$ into a Taylor series

$$\tilde{\mathbf{I}}(k) = \sum_{n=0}^{N} \mathbf{m}_n (k - k_0)^n \tag{6}$$

where k_0 is the expansion point, $\mathbf{m}_n$ denotes the unknown coefficients, and N denotes the total number of such coefficients. Substituting this into (3), one can obtain

$$\tilde{\mathbf{Z}}(k_0)\mathbf{m}_0 = \tilde{\mathbf{V}}(k_0) \tag{7}$$

$$\tilde{\mathbf{Z}}(k_0)\mathbf{m}_n = [\frac{\tilde{\mathbf{V}}^{(n)}(k_0)}{n!} - \sum_{i=1}^{n} \frac{\tilde{\mathbf{Z}}^{(i)}(k_0)\mathbf{m}_{n-i}}{i!}]. \tag{8}$$

To improve the computational efficiency, substitute (4) and (5) into (8) and carry out the associative law, one can obtain

$$\tilde{\mathbf{Z}}(k_0)\mathbf{m}_n = \mathbf{W}[\frac{\mathbf{V}^{(n)}(k_0)}{n!} - \sum_{i=1}^{n} \frac{\mathbf{Z}^{(i)}(k_0)(\mathbf{W}^H \mathbf{m}_{n-i})}{i!}] \ . \tag{9}$$

Then the multiplications between matrices in Eq.4 degenerated to be multiplications between matrixes and vectors. Computing the coefficients $\mathbf{m}_n$ $(n = 0, 1, 2, \cdots N)$ by iterative solvers, one can easily obtain $\tilde{\mathbf{I}}(k)$ in the given frequency band from Eq.6. Then the wavelet inverse transform is applied to $\tilde{\mathbf{I}}(k)$ ($\mathbf{I}(k) = \mathbf{W}^H \mathbf{I}(k)$); the electric current distribution $\mathbf{I}(k)$ and the radar cross section over a wide band will be obtained.

For a given threshold value, $\tilde{\mathbf{Z}}(k_0)$ will become a sparse matrix, namely, Eq.7 and Eq.9 become sparse matrix equations, which can be efficiently solved by a sparse solver.

Since the Taylor expansion has a limited bandwidth, $\tilde{\mathbf{I}}(k)$ can be represented with a better-behaved rational Padé function [2],

$$\tilde{\mathbf{I}}(k) = \frac{\sum_{i=0}^{L} \mathbf{a}_i (k - k_0)^i}{\sum_{j=0}^{M} \mathbf{b}_j (k - k_0)^j} \tag{10}$$

where $L + M = N$, $L = M$ or $M + 1$, and $b_0 = 1$. The unknown coefficients can be calculated by substituting (6) into (10). Multiplying (10) by the denominator of the Padé expansion and matching the coefficients of the equal powers of $k - k_0$, leads to the matrix equation

$$\begin{bmatrix} \mathbf{m}_L & \mathbf{m}_{L-1} & \mathbf{m}_{L-2} & \cdots & \mathbf{m}_{L-M+1} \\ \mathbf{m}_{L+1} & \mathbf{m}_L & \mathbf{m}_{L-1} & \cdots & \mathbf{m}_{L-M+2} \\ \mathbf{m}_{L+2} & \mathbf{m}_{L+1} & \mathbf{m}_L & \cdots & \mathbf{m}_{L-M+3} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{m}_{L+M-1} & \mathbf{m}_{L+M-2} & \mathbf{m}_{L+M-3} & \cdots & \mathbf{m}_L \end{bmatrix} \begin{bmatrix} \mathbf{b}_1 \\ \mathbf{b}_2 \\ \mathbf{b}_3 \\ \vdots \\ \mathbf{b}_M \end{bmatrix} = - \begin{bmatrix} \mathbf{m}_{L+1} \\ \mathbf{m}_{L+2} \\ \mathbf{m}_{L+3} \\ \vdots \\ \mathbf{m}_{L+M} \end{bmatrix}, \tag{11}$$

$$\begin{bmatrix} \mathbf{a}_0 \\ \mathbf{a}_1 \\ \mathbf{a}_2 \\ \vdots \\ \mathbf{a}_L \end{bmatrix} = \begin{bmatrix} \mathbf{m}_0 & 0 & 0 & \cdots & 0 \\ \mathbf{m}_1 & \mathbf{m}_0 & 0 & \cdots & 0 \\ \mathbf{m}_2 & \mathbf{m}_1 & \mathbf{m}_0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{m}_L & \mathbf{m}_{L-1} & \mathbf{m}_{L-2} & \cdots & \mathbf{m}_{L-M} \end{bmatrix} \begin{bmatrix} 1 \\ \mathbf{b}_1 \\ \mathbf{b}_2 \\ \vdots \\ \mathbf{b}_M \end{bmatrix}. \tag{12}$$

Fast lifting wavelet transform scheme

In discrete wavelet transform (DWT), the wavelet matrix $\mathbf{W}$ can be constructed by wavelet filter coefficients [8].

However, traditional implementation method caused auxiliary memories consumed by wavelet matrices, while operating wavelet using lifting scheme can avoid this limitation.

In the lifting scheme, we don't need to construct a wavelet matrix $\mathbf{W}$, but to operate the impedance matrix itself directly.

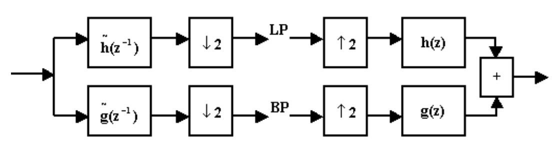

Fig. 1. Filter bank for wavelet transform.

The finite filter wavelet transform can be viewed as subband transform using finite impulse response (FIR) filters illustrated in Fig. 1. Forward transform uses two analysis filters $\tilde{h}$ (low pass) and $\tilde{g}$ (high pass), followed by subsampling, while inverse transform first upsamples and then uses two synthesis filters h (low pass) and g

(high pass). The perfect reconstruction (PR) property is defined by Eq. (13) [6],

$$\begin{aligned}\tilde{h}(z^{-1})h(z)+\tilde{g}(z^{-1})g(z)=2\\ \tilde{h}(-z^{-1})h(z)+\tilde{g}(-z^{-1})g(z)=0\end{aligned} \tag{13}$$

where z^{-1} in analysis filters is time reversion that compensates the delays in filters.

The polyphase representation of filter h, is given by

$$h(z)=h_e(z^2)+z^{-1}h_o(z^2) \tag{14}$$

where $h_e(z)=\sum_k h_{2k}z^{-k}$ contains the even coefficients, $h_o(z)=\sum_k h_{2k+1}z^{-k}$.

Define the new polyphase matrices

$$P(z)=\begin{bmatrix} h_e(z) & g_e(z)\\ h_o(z) & g_o(z)\end{bmatrix}, \tag{15a}$$

$$\tilde{P}(z)=\begin{bmatrix} \tilde{h}_e(z) & \tilde{g}_e(z)\\ \tilde{h}_o(z) & \tilde{g}_o(z)\end{bmatrix}. \tag{15b}$$

Then the PR condition can be rewritten as

$$P(z)\tilde{P}(z^{-1})^H=u \tag{16}$$

where u is an identity matrix.

The problem of finding an FIR wavelet transform thus amounts to finding a matrix $P(z)$. Once we have such a matrix, $\tilde{P}(z)$ and other filters for the wavelet transforms follow immediately. From (16) it follows that

$$\tilde{h}_e(z)=g_o(z^{-1}),\quad \tilde{h}_o(z)=-g_e(z^{-1}), \tag{17a}$$

$$\tilde{g}_e(z)=-h_o(z^{-1}),\ \tilde{g}_o(z)=h_e(z^{-1}). \tag{17b}$$

For the transforms with Daubechies wavelets, h_i and g_i are the coefficients involved in the two-scale relations of the Daubechies wavelets:

$$\phi(x)=\sqrt{2}\sum_{n=0}^{2N_m-1}h_n\phi(2x-n)\ , \tag{18a}$$

$$\psi(x)=\sqrt{2}\sum_{n=0}^{2N_m-1}g_n\phi(2x-n) \tag{18b}$$

where ϕ and ψ are the scaling and wavelet functions, respectively. N_m is the number of vanishing moments.

Daubechies has proved that given a complementary filter pair $\{h,g\}$ or $\{\tilde{h},\tilde{g}\}$, then there always exist Laurent polynomials $s_i(z)$ and $t_i(z)$ for $1\le i\le m$ and a nonzero constant K so that

$$P(z)=\prod_{i=1}^{m}\begin{bmatrix}1 & s_i(z)\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ t_i(z) & 1\end{bmatrix}\begin{bmatrix}K & 0\\ 0 & 1/K\end{bmatrix}, \tag{19a}$$

$$\tilde{P}(z)=\prod_{i=1}^{m}\begin{bmatrix}1 & 0\\ -s_i(z^{-1}) & 1\end{bmatrix}\begin{bmatrix}1 & -t_i(z^{-1})\\ 0 & 1\end{bmatrix}\begin{bmatrix}1/K & 0\\ 0 & K\end{bmatrix}. \tag{19b}$$

We can get inverse wavelet transform factoring formulation by simply inverse the forward formulation, switch additions and subtractions, and switch multiplications and divisions,

$$P^{-1}(z)=\prod_{i=m}^{1}\begin{bmatrix}1/K & 0\\ 0 & K\end{bmatrix}\begin{bmatrix}1 & 0\\ -t_i(z) & 1\end{bmatrix}\begin{bmatrix}1 & -s_i(z)\\ 0 & 1\end{bmatrix}, \tag{20a}$$

$$\tilde{P}^{-1}(z)=\prod_{i=m}^{1}\begin{bmatrix}K & 0\\ 0 & 1/K\end{bmatrix}\begin{bmatrix}1 & t_i(z^{-1})\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ s_i(z^{-1}) & 1\end{bmatrix}. \tag{20b}$$

We also describe lifting step by predict step and update step, which can be outlined in the following three basic operations.

Split: Divide the original data ($\mathbf{x}[n]$) into odd subsets ($\mathbf{x}_o[n]$) and even subsets ($\mathbf{x}_e[n]$),

$$\mathbf{x}_o[n]=\mathbf{x}[2n-1],\ \mathbf{x}_e[n]=\mathbf{x}[2n]. \tag{21a}$$

Predict: Generate high frequency component $\mathbf{d}(n)$ as the error in predicting odd subsets from even subsets using prediction operator Q,

$$d[n]=x_o[n]-Q(x_e[n]). \tag{21b}$$

Update: Generate low frequency component $\mathbf{c}[n]$ as a coarse similarity to original signal by applying an update operator U to the wavelet coefficients and adding to even subsets,

$$\mathbf{c}[n] = \mathbf{x}_e[n] + U(\mathbf{d}[n]) . \quad (21c)$$

The operators Q and U are decided by the polyphase matrixes.

To illuminate the steps for the fast lifting wavelet transform scheme, a D4 wavelet example is presented here.

The h and g filters are given by:

$$h(z) = h_0 + h_1 z^{-1} + h_2 z^{-2} + h_3 z^{-3}, \quad (22a)$$

$$g(z) = -h_3 z^2 + h_2 z^1 - h_1 + h_0 z^{-1} \quad (22b)$$

with $h_0 = \frac{1+\sqrt{3}}{4\sqrt{2}}$, $h_1 = \frac{3+\sqrt{3}}{4\sqrt{2}}$, $h_2 = \frac{3-\sqrt{3}}{4\sqrt{2}}$, and $h_3 = \frac{1-\sqrt{3}}{4\sqrt{2}}$.

The polyphase matrix is

$$P(z) = \begin{bmatrix} h_0 + h_2 z^{-1} & -h_3 z^1 - h_1 \\ h_1 + h_3 z^{-1} & h_2 z^1 + h_0 \end{bmatrix} \quad (23)$$

and the factorization is given by

$$P(z) = \begin{bmatrix} 1 & -\sqrt{3} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{\sqrt{3}}{4} + \frac{\sqrt{3}-2}{4} z^{-1} & 1 \end{bmatrix} \times \begin{bmatrix} 1 & z \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{\sqrt{3}+1}{\sqrt{2}} & 0 \\ 0 & \frac{\sqrt{3}-1}{\sqrt{2}} \end{bmatrix}.$$

Forward row transforms ($\mathbf{Z}(k)\mathbf{W}^H$) for impedance matrix $\mathbf{Z}(k)$ will be given by $P(z)$:

Set $\mathbf{x}$ to be one row of impedance matrix $\mathbf{Z}(k)$, $\tilde{\mathbf{x}}$ to be one row of impedance matrix $\tilde{\mathbf{Z}}(k)$ correspondingly.

Step1 (Split):

$$\mathbf{d}^{(0)}[n] = \mathbf{x}[2n-1], \mathbf{c}^{(0)}[n] = \mathbf{x}[2n].$$

Step2 (Predict):

$$\mathbf{d}^{(1)}[n] = \mathbf{d}^{(0)}[n] - \sqrt{3}\mathbf{c}^{(0)}[n].$$

Step3 (Update):

$$\mathbf{c}^{(1)}[n] = \mathbf{c}^{(0)}[n] + \frac{\sqrt{3}}{4}\mathbf{d}^{(1)}[n] + \frac{\sqrt{3}-2}{4}\mathbf{d}^{(1)}[n-1].$$

Then repeat step2 and step3:

$$\mathbf{d}^{(2)}[n] = \mathbf{d}^{(1)}[n] + \mathbf{c}^{(1)}[n+1] \quad ,$$

$$\tilde{\mathbf{x}}[2n] = \frac{\sqrt{3}+1}{2}\mathbf{c}^{(1)}[n] \quad ,$$

$$\tilde{\mathbf{x}}[2n-1] = \frac{\sqrt{3}-1}{2}\mathbf{d}^{(2)}[n] \quad .$$

Similarly, the forward column transforms ($\mathbf{WZ}(k)$ or $\mathbf{WV}(k)$) for impedance matrix $\mathbf{Z}(k)$ will give by $\tilde{P}(z)$ which can be computed by Eq.17 and Eq.15b; the inverse current vector transform $\mathbf{W}^H\tilde{\mathbf{I}}(k)$ will be operated by $\tilde{P}^{-1}(z)$. The transform $\mathbf{Z}(k)\mathbf{W}$ is not needed in this paper, and it can be operated by $P^{-1}(z)$.

III. NUMERICAL RESULTS

To validate the analysis presented in the previous sections, a few numerical examples are considered. For perfectly conducting infinite objects excited by a TM plane wave at an angle of incident θ_i , RCS calculations over a frequency band are done for a cylinder with perimeter $C = 0.36$ m ($\theta_i = 0$), a square cylinder with length $a = 0.1$ m ($\theta_i = 0$), and a strip with length w =0.25m and width $d = 0.001$m ($\theta_i = \pi/4$). In the numerical examples presented below, the expansion frequency is chosen to be the center frequency of the band of interest. Fig.2 shows the nonzero components distribution of the impedance matrix after transforms. The results over a given frequency band are calculated by the LWT-AWE method with Padé approximation ($L = 4$, $M = 3$).

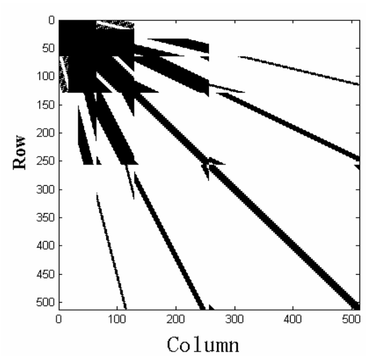

Fig. 2. Impedance matrix of cylinder after transforms.

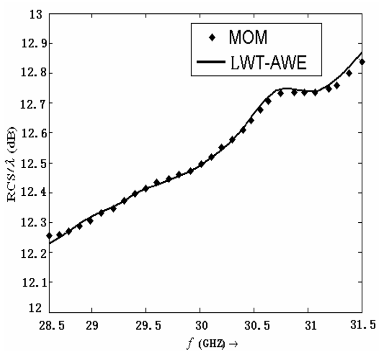

Fig. 3. RCS frequency response of the cylinder.

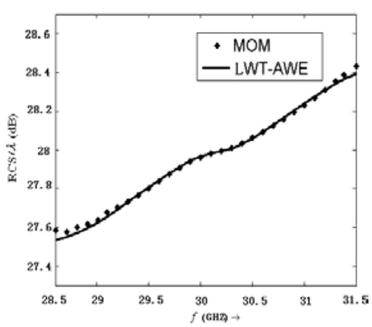

Fig. 4. RCS frequency response of the square cylinder.

The CPU time consumed by LWT-AWE and the moment methods are given in Table I. All the computations reported are done on a PIV 2.66G/256MB computer.

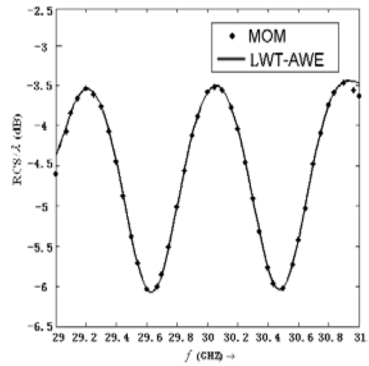

Fig. 5. RCS frequency response of the strip.

Table I. CPU time required comparison.

Examples	Figure 3 time(s)	Figure 3 frequencies	Figure 5 time(s)	Figure 5 frequencies
MOM	763.7	31	947.4	41
LWT-AWE	194.2	301	279.1	401

IV. CONCLUSIONS

An implementation of AWE combined with LWT for frequency-domain MOM is presented. The RCS for different PEC objects are computed. From the numerical examples presented, LWT-AWE method is found to be superior in terms of the CPU time to obtain a frequency response: CFIE eliminates interior resonance problem, and the employment of LWT produces a sparse system of linear equations that are treated effectively by a sparse linear system solver.

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