Chinese Journal of Electronics
Vol.15, No.4, Oct. 2006

Chebyshev Approximation for Fast Frequency-Sweep Analysis of Electromagnetic Scattering Problem*

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Abstract — To complete the scattering analysis of an arbitrary shaped perfectly electric conductor over a wide frequency band, the Chebyshev polynomial of first kind is applied. The Chebyshev nodes within a given frequency range are found, and then the surface electric currents at these nodes are computed by the method of moments. The surface current is expanded in a polynomial function via the Chebyshev approximation. Using this function, the electric current distribution can be obtained at any frequency within the given frequency range. The numerical results are compared with the results obtained by the method of moments, and the complexity of computation is reduced obviously.

Key words — Method of moments, Chebyshev approximation, Wide-band radar cross section.

I. Introduction

For many problems in electromagnetics, knowledge of broadband response of scattering objects is required. For years, the Method of moments (MOM) is one of the most popular techniques for scattering analysis in frequency domain[1]. By the conventional approach, to obtain the wide-band response with MOM, a set of algebraic equations must be solved repeatedly at a number of frequency samples. If the Radar cross section (RCS) is highly frequency dependent, one needs to do the calculations at finer increments of frequency to get an accurate representation of the frequency response. Consequently, the CPU time needed becomes unacceptably long. To overcome this difficulty, the approximate techniques that can efficiently simulate frequency response over a wide band with acceptable computational time are desired.

Over past few years, the Asymptotic waveform evaluation (AWE) technique is one of the leading methods based on MOM to obtain the wide-band RCS[2,3]. The AWE method is found to be superior in terms of the CPU time to obtain frequency response. However, the expected effect frequency band is limited by the inherent property of the Taylor series, and the memory needed is greatly increased on account of the high derivatives of the dense impedance matrix.

It is well-known that the Chebyshev series can be used to approximate a function at least as accurately as a polynomial of the same degree similarly obtained from a series in any other orthogonal polynomials, in other words, the Chebyshev expansion is as good as the best polynomial approximations[4,5]. Recently, the Chebyshev approximation theory is widely applied to the design of antennas and other fields in electromagnetism[6].

In this paper, the application of Chebyshev approximation theory for predicting the RCS over a band of frequencies using MOM is described. The Chebyshev nodes for a given frequency band are obtained firstly by coordinate transform, and the surface electric currents at these nodes are computed by the method of moments to get the Chebyshev series coefficients. Using the polynomial function, the surface current is obtained at any frequency within the given frequency range, which is used to calculate the RCS. Numerical results for a finite cylinder, a cube, and a sphere are presented in Section III. The numerical data is compared with the results obtained by the method of moments, the Chebyshev method is found to be superior in terms of the CPU time to obtain a frequency response without sacrificing much memory.

II. Theory

1. Method of moments

Consider an arbitrary shaped Three-dimensional (3D) Perfectly electric conductor (PEC) body illuminated by a plane wave. The total field is a combination of the incident field (labeled by superscript i) and the scattered field (labeled by superscript s), *i.e.*

$$\boldsymbol{E} = \boldsymbol{E}^s + \boldsymbol{E}^i \tag{1}$$

The scattered electric field $\boldsymbol{E}^s$ is due to surface currents and free charges on the metal surface S

$$\boldsymbol{E}^s = -j\omega \boldsymbol{A}_S(\boldsymbol{r}) - \nabla \Phi_S(\boldsymbol{r}), \boldsymbol{r} \text{ on } S \tag{2}$$

where $\boldsymbol{A}_S(\boldsymbol{r})$ is the magnetic vector potential, and $\Phi_S(\boldsymbol{r})$ denotes the scalar potential.

*Manuscript Received Nov. 2005; Accepted May 2006. This work is supported by the National Natural Science Foundation of China (No.60371041).

On the surface of a PEC body, the tangential component of the electric field $\boldsymbol{E}_{\tan} = 0$, thus giving Electric field integral equation (EFIE),

$$\boldsymbol{E}^i_{\tan} = (j\omega \boldsymbol{A}_S + \nabla \Phi_S)_{\tan}, \boldsymbol{r} \text{ on } S \quad (3)$$

Utilizing the Rao-Wilton-Glisson (RWG) basis functions[7], the surface current density $\boldsymbol{J}_S$ is expanded into basis function in the form

$$\boldsymbol{J}_S = \sum_{n=1}^{N} I_n \boldsymbol{f}^S_n \quad (4)$$

where $\boldsymbol{f}^S_n$ is the vector basis function defined in Ref.[7].

Then the EFIE is reduced to a matrix equation:

$$Z(k)I(k) = V(k) \quad (5)$$

where k is the wave number, $Z(k)$ is an $N \times N$ impedance matrix, and $V(k)$ is a N dimensional voltage vector, with their elements defined as

$$Z(k)_{mn} = \left(\frac{j\omega\mu}{4\pi}\right) \int_S \int_S \boldsymbol{f}^S_m(\boldsymbol{r}) \bullet \boldsymbol{f}^S_n(\boldsymbol{r}') g dS dS' - \left(\frac{j}{4\pi\omega\varepsilon}\right) \int_S \int_S (\nabla_S \bullet \boldsymbol{f}^S_m) \bullet (\nabla_S \bullet \boldsymbol{f}^S_n) g dS dS' \quad (6)$$

$$V(k)_m = \int_S \boldsymbol{f}^S_m \bullet \boldsymbol{E}^i dS \quad (7)$$

where $g = e^{-jk|\boldsymbol{r}-\boldsymbol{r}'|}/|\boldsymbol{r}-\boldsymbol{r}'|$, is the free-space Green's function.

2. The Chebyshev approximation theory

Since the electric current in Eq.(5) is calculated at a single frequency, to obtain the RCS over a wide band, one may need to solve Eq.(5) repeatedly. Here the Chebyshev approximation theory is applied to the scattering analysis of objects within a given frequency range.

Theorem 1 For $H_n = span\{1, x, x^2, \cdots, x^n\}$, $P_n(x) \in H_n$, $f(x) \in C[a,b]$, if there exists $n+2$ points $a \le x_1 \le x_2 \le \cdots x_{n+2} \le b$ satisfy

$$P(x_l) - f(x_l) = (-1)^l \sigma \|P(x) - f(x)\|_\infty \quad (8)$$

where $\sigma = \pm 1$, then $P(x)$ will be the best polynomial approximations in H_n for $f(x)$[8].

Definition 1 The Chebyshev polynomial of degree n is denoted as $T_n(x)$, which is given by the recursive relations:

$$T_0(x) = 1, \quad T_1(x) = x, \quad T_{n+1}(x) = 2xT_n(x) - T_{n-1}(x), \quad n \ge 1 \quad (9)$$

And the Chebyshev approximation formula for $f(x) \in C[-1,1]$ is given by

$$f(x) = -\frac{1}{2}c_0 + \sum_{l=0}^{\infty} c_l T_l(x) = P_n(x) + \sum_{l=n+1}^{\infty} c_l T_l(x), \quad x \in [-1,1] \quad (10)$$

Hence $f(x) - P_n(x) \approx c_{n+1}T_{n+1}(x)$, since for $T_{n+1}(x)$, there are $n+2$ points

$$x_l = \cos\left(\frac{l\pi}{n+1}\right), \quad l = 0, 1, \cdots, n+1$$

satisfy Eq.(8), so the Chebyshev series can be as good as the best polynomial approximations.

Then the scattering analysis over a wideband frequency is considered. For a given frequency range $f \in [f_a, f_b]$, $k \in [k_a, k_b]$, the coordinate transform is used as

$$\tilde{k} = [2k - (k_a + k_b)]/(k_b - k_a) \quad (11)$$

The electric current $I(k)$ will be calculated by

$$I(k) = I\left(\frac{\tilde{k}(k_b - k_a) + (k_b + k_a)}{2}\right) \quad (12)$$

and the Chebyshev approximation for $I(k)$ is given by

$$I(k) = I\left(\frac{\tilde{k}(k_b - k_a) + (k_b + k_a)}{2}\right) \approx \sum_{l=0}^{n-1} c_l T_l(\tilde{k}) - \frac{c_0}{2} \quad (13)$$

$$c_l = \frac{2}{n} \sum_{i=1}^{n} I(k_i) T_l(\tilde{k}_i) \quad (14)$$

where $\tilde{k}_i (i = 1, 2, \cdots, n)$ are the Chebyshev nodes for $T_n(\tilde{k})$, and $k_i \in [k_a, k_b]$ can be obtained by

$$k_i = \frac{\tilde{k}_i(k_b - k_a) + (k_a + k_b)}{2} \quad (15)$$

and the surface electric current $I(k_i)$ can be obtained by method of moments.

III. Numerical Results

To validate the analysis presented in the previous section, a few numerical examples are considered.

The first example is for a finite PEC cylinder that illuminated by a plane wave $\boldsymbol{E}^i = \boldsymbol{e}_x e^{-jk_z z}$, the diameter of the cylinder is 1cm with its height 2cm. The cylinder is discretized into 468 triangular elements resulting in 702 unknown current coefficients. The RCS frequency response is shown in Fig.1. There are 5 Chebyshev nodes chosen for the frequency band 5GHz to 30GHz.

As a second example, the RCS response of a PEC cube (1cm×1cm×1cm) is calculated over a frequency band (2GHz∼22GHz). The cube is illuminated by a plane wave $\boldsymbol{E}^i = \boldsymbol{e}_x e^{-jk_z Z}$. The cube is discretized with 320 triangular subdomains resulting in 480 current unknown coefficients. The RCS frequency response of it is shown in Fig.2, and there are 6 Chebyshev nodes chosen for the given frequency range.

Finally, a PEC sphere of radius 0.4cm is considered. The sphere is illuminated by a plane wave $\boldsymbol{E}^i = \boldsymbol{e}_x e^{-jk_z Z}$. The object is discretized into 408 triangular elements that result in 612 unknown current coefficients. As shown in Fig.3, the RCS of the sphere is compared with the results computed by MOM and Mie series, and there are 7 Chebybshev nodes chosen for the given frequency band.

Table 1. CPU time comparison (seconds)

Method / Example	MOM		Chebyshev	
	CPU time	Frequency points	CPU time	Frequency points
Fig.1	2414.71	26	484.23	251
Fig.2	1647.83	41	264.53	401
Fig.3	2103.54	41	371.69	401

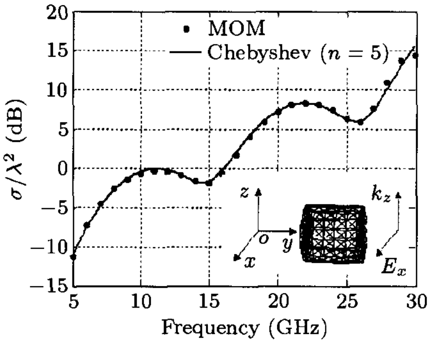

Fig. 1. The RCS frequency response of the cylinder

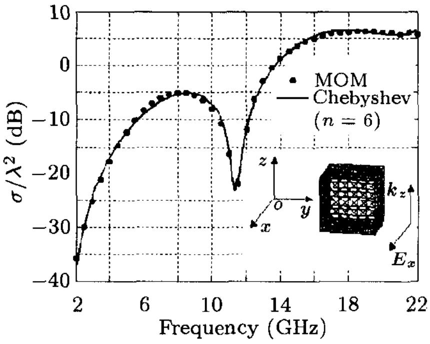

Fig. 2. The RCS frequency response of the cube

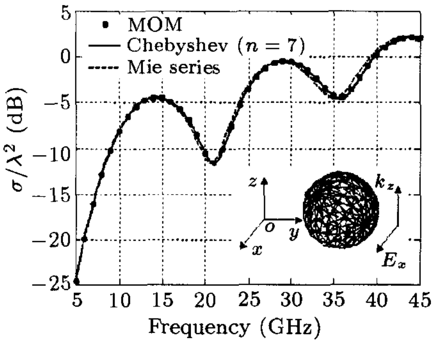

Fig. 3. The RCS frequency response of the sphere

The CPU time for RCS calculation and frequency points computed are given in Table 1; all the computations reported are done on a PIV 2.66G/1GMB computer.

IV. Conclusion

An implementation of Chebyshev approximation method for MOM is presented. The RCS for different PEC objects are computed and compared with the traditional method over a wide frequency band, the Chebyshev method is found to be superior in terms of the CPU time to obtain a frequency response without sacrificing much memory. The accuracy of Chebyshev method over a desired frequency band and its relation to the order of Chebyshev series to be chosen is topics of interest for future research. With these topics addressed, Chebyshev approximation theory will be of good application in obtaining the frequency response using a frequency-domain technique such as MOM.

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