Electromagnetic Effective Degree of Freedom of an MIMO System in Free Space

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Abstract—Effective degree of freedom (EDOF) of a multiple-input–multiple-output (MIMO) system represents its equivalent number of independent single-input–single-output (SISO) systems, which directly characterizes the communication performance. Traditional EDOF only considers single polarization, where the full polarized components degrade into two independent transverse components under the far-field approximation. However, the traditional model is not applicable to complex scenarios especially for the near-field region. Based on an electromagnetic (EM) channel model built from the dyadic Green's function, we first calculate the EM EDOF to estimate the performance of an arbitrary MIMO system with full polarizations in free space. Then, we clarify the relations between the limit of EDOF and the optimal number of sources/receivers. Finally, potential benefits of near-field MIMO communications are demonstrated with the EM EDOF, in which the contribution of the longitudinally polarized source is taken into account. This letter establishes a fundamental EM framework for MIMO wireless communications.

Index Terms—Dyadic Green's function, effective degree of freedom (EDOF), multiple-input–multiple-output (MIMO), near-field communications.

I. INTRODUCTION

BASED on the Shannon's information theory [1], multiple-input–multiple-output (MIMO) technology using spatial multiplexing has been developed for enhancing the channel capacity of modern wireless communications [2], [3]. Different electromagnetic (EM) modes, as orthogonal bases, have been employed in MIMO systems, like conventional plane-wave modes and newly introduced orbital angular momentum modes [4]–[6]. The sources/receivers in an MIMO system include antenna arrays [3] and metasurfaces [7], [8], represented by the discrete and quasi-continuous EM sources.

As an important concept in information theory, effective degree of freedom (EDOF) of an MIMO system represents its equivalent number of independent single-input–single-output (SISO) systems [9], [10], which conveniently estimates the performance of the MIMO system through a single number. Compared to the concept of degree of freedom (DOF), which is defined as the number of significant singular values (eigenvalues) of the channel (correlation) matrix, the EDOF is directly related to the slope of spectral efficiency [11], and, thus, to the channel performance. The EDOF of an MIMO system, as well as its limit, needs to be estimated with the channel matrix [12], [13]. Typically, the channel matrix is modeled based on the scalar Green's function for short-range MIMO in free space [14]–[17], or some approximation methods tailored to multipath environments [10], [18]. However, these models either break down at the near-field region or cannot capture the full-wave physics (full polarizations) of EM fields.

As EM wave is the physical carrier of information, the DOF of MIMO system has also been investigated from EM perspectives [19], [20]. Mathematically, two groups of infinite orthogonal modes are defined in a Hilbert space to represent the source and radiation fields, then the DOF limit is deduced with functional methods [21], [22]. To find the ultimate capacity of an MIMO antenna system [23], the surface currents of the antenna elements are expanded as the transmitting (source) bases. Furthermore, an interesting model for the hologram MIMO has been proposed recently [24], where the channel matrix is built with plane-wave expansion. Additionally, the DOF of an MIMO system is proved as the bound of its EDOF through EM methods [25]. Nevertheless, clear definition and detailed discussion of the EM EDOF of an MIMO system, as well as its connection with the EM DOF, have not been explored yet.

Three contributions are made in this letter. First, with the aid of a mathematical definition of EDOF from information theory, the EM EDOF is defined based on an EM channel matrix deduced from the dyadic Green's function. Second, the DOF and EDOF limit of a space-constrained MIMO system are discussed. Third, we demonstrate the potential benefits of near-field MIMO communications with the consideration of the longitudinally polarized current source. These comprehensive and comparative studies have not been investigated before.

II. EM CHANNEL MODEL OF AN MIMO SYSTEM

Considering a basic MIMO model in free space, as depicted in Fig. 1(a), a set of N_S transmitting antennas at the positions
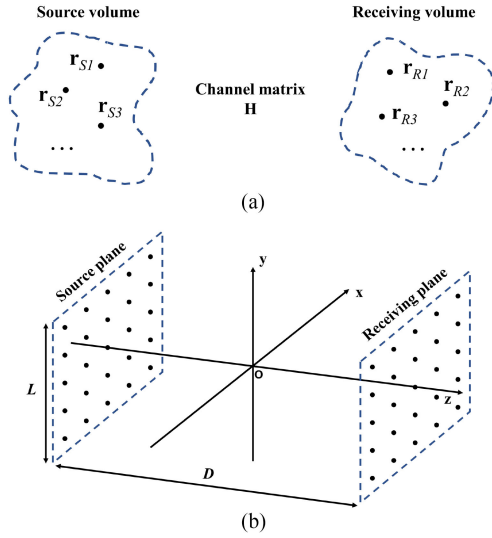
Fig. 1. (a) Basic model of an MIMO system in free space. Transmitting antennas at the positions $\mathbf{r}_{Sn}(n = 1, \ldots, N_S)$ are distributed in the source volume and receiving antennas at the positions $\mathbf{r}_{Rm}(m = 1, \ldots, N_R)$ are distributed in the receiving volume. The transmitting/receiving antennas in the two volumes are coupled by a channel matrix $\mathbf{H}$. (b) MIMO system consists of $N \times N$ uniformly distributed point sources/receivers on two identical square source/receiving planes. L is the side length of the planes, and D is the distance between the two planes.

$\mathbf{r}_{Sn}(n = 1, \ldots, N_S)$ are distributed in the source volume, N_R receiving antennas at the positions $\mathbf{r}_{Rm}(m = 1, \ldots, N_R)$ are distributed in the receiving volume. To make the following equations clear, the transmitting and receiving antennas are modeled as isotropic point sources/receivers (delta function basis), which is the widely used assumption made in EM information theory [26]. From EM perspective, the antennas can also be modeled as continuous surface (equivalent) currents by rooftop or Rao–Wilton–Glisson (RWG) basis, as frequently utilized in the methods of moments [27]. Different basis representations of the currents, which relates to different antenna designs, will not influence the estimations of the EDOF and DOF limit. Therefore, the mutual coupling between antenna elements is ignored here to draw fundamental physical conclusions and explore insightful engineering rules.

A. Model Based on Scalar Green's Function

For a set of N_S point sources at the positions $\mathbf{r}_{Sn}$ in the source volume with the complex amplitudes t_n, as shown in Fig. 1(a), the superposed electric field at the positions $\mathbf{r}_{Rm}$ in the receiving volume would be

$$E(\mathbf{r}_{Rm}) = \frac{1}{4\pi} \sum_{n=1}^{N_S} \frac{\exp(-jk_0 |\mathbf{r}_{Rm} - \mathbf{r}_{Sn}|)}{|\mathbf{r}_{Rm} - \mathbf{r}_{Sn}|} t_n = \sum_{n=1}^{N_S} h_{mn} t_n \tag{1}$$

where

$$h_{mn} = \frac{1}{4\pi} \frac{\exp(-jk_0 |\mathbf{r}_{Rm} - \mathbf{r}_{Sn}|)}{|\mathbf{r}_{Rm} - \mathbf{r}_{Sn}|} = g(\mathbf{r}_{Rm}, \mathbf{r}_{Sn}) \tag{2}$$

which is the scalar Green's function with certain source/receiver positions. The received signals at the point receivers would be

the sum of the fields from all the point sources added up at $\mathbf{r}_{Rm}$

$$f_m = \sum_{n=1}^{N_S} h_{mn} t_n \tag{3}$$

If t_n and f_m are collected in the two column vectors $t = [t_1, t_2, \ldots, t_{N_S}]^T$ and $f = [f_1, f_2, \ldots, f_{N_R}]^T$, we can define the projection from the point sources to the point receivers as

$$f = \mathbf{H}t \tag{4}$$

with

$$\mathbf{H} = \begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1N_S} \\ h_{21} & h_{22} & \cdots & h_{2N_S} \\ \vdots & \vdots & \ddots & \vdots \\ h_{N_R1} & h_{N_R2} & \cdots & h_{N_RN_S} \end{bmatrix} \tag{5}$$

which is the channel matrix based on the scalar wave equation. This model considering single polarization is sufficient for far-field communications, but not for near-field communications.

B. Model Based on Dyadic Green's Function

In order to model an arbitrary MIMO system with full polarizations, dyadic Green's function is utilized. Considering the same configuration as the previous subsection, we adopt three polarizations at each point source/receiver. The dyadic Green's function in free space is

$$\bar{\mathbf{G}}(\mathbf{r}, \mathbf{r}') = \left(\bar{\mathbf{I}} + \frac{\nabla\nabla}{k_0^2}\right) g(\mathbf{r}, \mathbf{r}') \tag{6}$$

where $\bar{\mathbf{I}}$ is the unit tensor. The tensor $\bar{\mathbf{G}}$ could be rewritten as a matrix form, i.e.,

$$\bar{\mathbf{G}} = \begin{bmatrix} G_{xx} & G_{xy} & G_{xz} \\ G_{yx} & G_{yy} & G_{yz} \\ G_{zx} & G_{zy} & G_{zz} \end{bmatrix} \tag{7}$$

where each element is a scalar Green's function between one polarization of field and one polarization of source, denoted by its subscript.

As discussed in the previous subsection, an $N_R \times N_S$ channel matrix is built for an MIMO system with one scalar Green's function. The three polarizations of electric field are orthogonal, so that we could write the channel of full polarizations in a matrix form without loss of information. Considering (4) in the full-polarization case, the complex amplitudes of sources $t = [t_x t_y t_z]^T = [t_{x1}, \ldots, t_{xN_S}, t_{y1}, \ldots, t_{yN_S}, t_{z1}, \ldots, t_{zN_S}]^T$ is a $3N_S \times 1$ vector, the complex amplitudes of received signals $f = [f_x f_y f_z]^T = [f_{x1}, \ldots, f_{xN_R}, f_{y1}, \ldots, f_{yN_R}, f_{z1}, \ldots, f_{zN_R}]^T$ is a $3N_R \times 1$ vector, where the x, y, z in subscripts denote the polarizations and the $1, 2, \ldots, N$ in subscripts denote the positions. The two column vectors are then related by a $3N_R \times 3N_S$ EM channel matrix

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}_{xx} & \mathbf{H}_{xy} & \mathbf{H}_{xz} \\ \mathbf{H}_{yx} & \mathbf{H}_{yy} & \mathbf{H}_{yz} \\ \mathbf{H}_{zx} & \mathbf{H}_{zy} & \mathbf{H}_{zz} \end{bmatrix} \tag{8}$$

where the nine $N_R \times N_S$ matrices correspond to the nine scalar Green's functions. This matrix contains all the information

needed for an MIMO system in free space, and the model could be extended to arbitrary expansion bases and propagation environments. Notice that the correlation matrix $\mathcal{R} = \mathbf{H}\mathbf{H}^\dagger$ or $\mathbf{H}^\dagger\mathbf{H}$ sharing the same eigenvalues is used for estimating the performance of an MIMO system, as the receiving (transmitting) power is related by $\bar{\mathbf{G}}^\dagger\bar{\mathbf{G}}$ ($\bar{\mathbf{G}}\bar{\mathbf{G}}^\dagger$) [22].

III. EM EDOF OF AN MIMO SYSTEM

A. Definition of EM EDOF

The concept of the EDOF of an MIMO system originates from information theory [9]–[11], which indicates the spectral efficiency (channel capacity at a single frequency) of an MIMO system through a scalar number. Traditional EDOF is calculated with (5), thus cannot be applicable to the near-field regime. The EM EDOF introduced here utilizes the same mathematical operation, but the EM channel matrix, i.e., (8), incorporates full polarizations.

Considering that the power is equally allocated to identical antenna elements with fixed total signal-to-noise ratio (SNR) ρ, which is commonly adopted in typical MIMO applications. The capacity of an MIMO system can be calculated with

$$C = B \sum_{i=1}^{n} \log_2\left(1 + \frac{\rho}{n}\sigma_i\right) \quad (9)$$

where B is the antenna bandwidth, n is the rank (or DOF) of the correlation matrix $\mathcal{R}$, and σ_i are the corresponding eigenvalues. If the MIMO system is ideal ($\mathcal{R}$ is a unit matrix, all the subchannels are independent), (8) becomes

$$C = Bn \log_2\left(1 + \frac{\rho}{n}\right). \quad (10)$$

This ideal case is unreachable, because σ_i are different in practical. However, in reality, the channel capacity can still be written as a separated form

$$C \approx B\Psi_e \log_2\left(1 + \frac{\rho}{\Psi_e}\right) \quad (11)$$

where $\Psi_e \in [1, n]$ is the EDOF, i.e., the equivalent number of SISO systems, and is equal to n in ideal case. The concept and estimation of the EDOF have been discussed in [9]–[11], and Ψ_e can be approximately calculated as

$$\Psi_e(\mathcal{R}) = \left(\frac{\text{tr}(\mathcal{R})}{\|\mathcal{R}\|_F}\right)^2 = \frac{(\sum_i \sigma_i)^2}{\sum_i \sigma_i^2} \quad (12)$$

where $\text{tr}(\cdot)$ represents the trace operator and the subscript F denotes the Frobenius norm, this equation has been proved to be sufficiently accurate in estimating the capacity of a practical MIMO system, as seen in Fig. 2.

B. Limit of EM EDOF

In practical implementation, engineers are concerned about the limit of EM EDOF of a space-constrained MIMO system, and also the number of sources/receivers for approaching the limit. Intuitively, when the number of sources/receivers surpasses a certain saturation point, the exceeding eigenvalues, corresponding to the exceeding sources/receivers, will become too small to contribute to the EM EDOF, and the EM EDOF
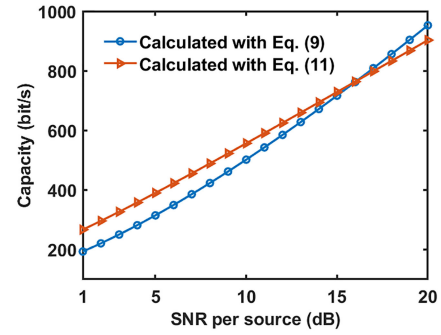

Fig. 2. Comparison between the capacities calculated with (9) and (11). 20×20 point sources/receivers are uniformly distributed on $10 \times 10\ \lambda_0^2$ source/receiving plane, the distance between the two planes is $7\ \lambda_0$, λ_0 is free-space wavelength, and bandwidth B is set as 1.
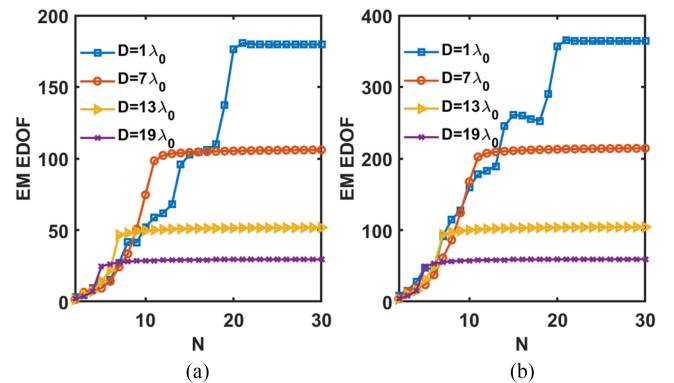

Fig. 3. EM EDOFs calculated with different N (number of sources/receivers in one side) and D (distance). Side length L is fixed as $10\ \lambda_0$. (a) Scalar Green's function. (b) Dyadic Green's function.

will reach its maximum value. With the proposed EM channel model, we can precisely estimate the limit of EM EDOF of an MIMO system.

Without loss of generality, we consider an MIMO system consisting of two $L \times L$ square planes separated by the distance D, with uniformly distributed $N \times N$ point sources/receivers on the source/receiver planes, as depicted in Fig. 1(b). To investigate the limit of EM EDOF, L is fixed as $10\ \lambda_0$, then the EM EDOFs with different N and distance D are calculated. The results calculated with scalar and dyadic Green's functions are presented in Fig. 3. Regarding the scalar Green's function case, the far-field E-field is polarized at x- or y-direction shown in Fig. 1(b). It can be observed that when N increases, the EM EDOF will increase fast in the beginning, then reach a maximum value in all the cases. An interesting phenomenon is that the EDOF appears a partial decrease at $D = 1\ \lambda_0$ in Fig. 3(b), while similar tendency does not appear in the scalar case. The reason is that the dyadic Green's function produces a rigorous wave-physics solution to capture the near-field evanescent waves with three polarized components.

C. Optimal Number of Sources/Receivers

Optimal number of sources/receivers is simultaneously estimated in the previous subsection, which is the number of significant eigenvalues (i.e., DOF) of the EM correlation matrix.
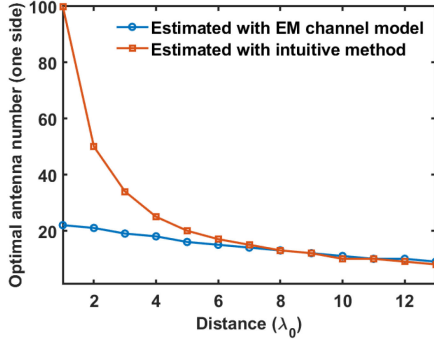
Fig. 4. Optimal number of sources/receivers calculated with the EM channel model and the intuitive method, with the side length $L = 10\,\lambda_0$ and distance $D = 1 - 13\,\lambda_0$.
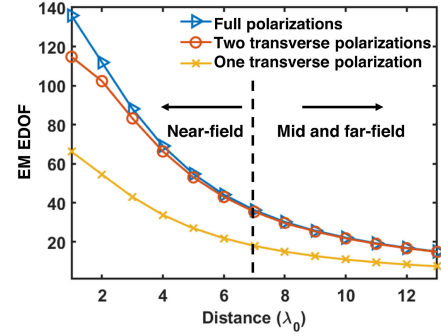

Fig. 5. EM EDOFs of MIMO systems calculated with different number of polarizations, the side length $L = 5\,\lambda_0$, antenna number $N = 11$ and distance $D = 1 - 13\,\lambda_0$.

Nonetheless, there are some elegant intuitive methods. Under the paraxial approximation in optics, i.e., the distance between the two planes is much larger than their sizes, an intuitive method from the perspective of solid angle is proposed to calculate the number of significant EM modes [28]

$$\Psi = \frac{A_S A_R}{\lambda_0^2 D^2} \tag{13}$$

where A_S is the area of source plane, A_R is the area of receiving plane, and D is the distance between the two planes. We compare the optimal number of sources/receivers calculated with the EM channel model and the intuitive method, with $L = 5\,\lambda_0$ and $D = 1 - 13\,\lambda_0$. As shown in Fig. 4, when the distance between the two planes becomes large, the two methods give almost the same results, indicating that the intuitive method is accurate at a relatively long distance. Equation (13) can also be extended to half-space isotropic multipath environment with a source plane and an ideal half-space receiving plane, also regarded as a Fourier spectral method [24], [29], [30]. The intuitive methods are only suitable for the MIMO systems at far field, while the EM channel model could be readily applicable to more complex scenarios.

The EM DOF of an MIMO system refers to the number of dominant EM modes (rank), thus it is identical to the optimal number of sources/receivers for approaching the limit of EM EDOF. The EM EDOF is rather a mathematical tool to denote the equivalent number of SISOs of an MIMO system, which is directly related to the spectral efficiency.

IV. Near-Field MIMO Communications

It is interesting to investigate the potential benefit of the near-field (reactive) MIMO communications compared to the far-field communications with the EM EDOF, which cannot be accomplished with the traditional method. Other than the enhancement of EM EDOF brought by the decrease of distance, the available polarizations of current sources are fundamentally different at near field and far field. Considering the dyadic Green's function with the far-field approximation $\nabla \to j\mathbf{k}_0$, (6) becomes

$$\bar{\mathbf{G}}^f(\mathbf{r}, \mathbf{r}') = (\bar{\mathbf{I}} - \mathbf{a_r a_r})\, g(\mathbf{r}, \mathbf{r}') \tag{14}$$

where $\mathbf{a_r a_r}$ is the r-polarized component of unit tensor in spherical coordinate, i.e., the z-polarized component in the Cartesian coordinate in Fig. 1(b). Obviously, the z-polarization is subtracted, which indicates that the z-polarization of current source will not contribute to the far-field communications. Therefore, only the two transverse polarizations of current sources are retained, the dyadic Green's function with the far-field approximation can be simplified to be

$$\bar{\mathbf{G}}^f = \begin{bmatrix} G_{xx} & G_{xy} & 0 \\ G_{yx} & G_{yy} & 0 \\ G_{zx} & G_{zy} & 0 \end{bmatrix} \tag{15}$$

where the third column denotes the contribution of the z-polarization.

In order to demonstrate the potential benefit of near-field MIMO communications, we calculate the EM EDOF based on the dyadic Green's functions with one transverse, two transverse and full polarizations, i.e., (15) and (7) for the latter two, respectively. With the side length $L = 5\,\lambda_0$, the regions of near field and far field can be calculated as $D_n = 0.62 \cdot (L^3/\lambda_0)^{1/2} = 6.9\,\lambda_0$ and $D_f = 2 \cdot L^2/\lambda_0 = 50\,\lambda_0$ [31]. The number of antennas is fixed as 11×11, with which the limit of EM EDOF is almost approached in all the cases. The contribution of the z-polarization of current source at near field is clearly illustrated in Fig. 5. It can be observed that the benefit brought by the z-polarization vanishes when the distance D exceeds the near-field range; the EM EDOF of full polarizations doubles that of single polarization at mid and far fields. Hence, the benefit of near-field MIMO communications from the z-polarization only appears at a relatively small distance in free space.

V. Conclusion

Based on the EM channel model of an MIMO system in free space, we discussed the principle of EM EDOF, limit of EM EDOF and the optimal number of sources/receivers. This method is useful for estimating the capacity limit of an MIMO system from EM perspective, and can also be utilized for optimizing array arrangement as well as analyzing near-field communication. The proposed method can be readily extended to arbitrary source/receiver types and propagating environments, by using proper basis functions [27] and numerical dyadic Green's function in arbitrary background [32].

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