Fast Frequency Sweep Scattering Analysis for Multiple PEC Objects

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Abstract **–** The best polynomial approximation, performed by Chebyshev approximation , is applied to the scattering analysis of multiple arbitrary shaped perfectly electric conducting objects over a broad frequency band. For a given frequency band, the frequency points corresponding to the Chebyshev nodes are found by transformation of coordinates, and the surface electric currents at these points are computed by the method of moments. The surface current is represented by a polynomial function via the Chebyshev approximation, and the electric current distribution can be obtained at any frequency point within the given frequency band. Numerical examples show that the results generated by the presented approach agree to that provided by the method of moments at each of the frequency points, but the CPU time of the presented approach is reduced obviously without sacrificing much memory.

Index Terms **–** Method of moments (MOM), the best polynomial approximation, broad band radar cross section, multiple PEC objects.

I. INTRODUCTION

In many radar applications it is necessary to determine the scattering from objects over a broad frequency band. The solution of the electric field integral equation (EFIE) via the method of moments (MOM) has been one of the most popular tools for accurately predicting the radar cross-section (RCS) of arbitrarily shaped perfect electric conducting (PEC) objects in the frequency domain [1]. In MOM, the integral equation is reduced to a matrix equation by dividing the PEC body surface into sub-domains and employing a dispersed method. The RCS is computed from the surface currents obtained from the matrix equation.

To complete the scattering analysis over a wide frequency band using MOM, one has to repeat the calculation at each of the frequency points over the band of interest. If the far-field is highly frequency dependent, one needs to do the calculation at finer increment of frequency to get an accurate representation of the frequency response, which must be computationally intensive. To overcome this difficulty, the approximate solution techniques that can efficiently simulate frequency response over a wide band with acceptable computational time are desired.

Recently, the asymptotic waveform evaluation (AWE) technique has been successfully used in various electromagnetic problems especially in obtaining the broad band RCS [2], [3]. In the AWE technique, a Taylor series expansion is generated to approximate the equivalent surface current. And the rational function approach is applied to improve the accuracy of the numerical solution. As compared with using MOM at each of the frequency points, the AWE method is found to be superior in terms of the CPU time to obtain frequency response. However, the accuracy of the Taylor series is limited by the radius of convergence, and the memory needed is greatly increased on account of the high derivatives of the dense impedance matrix.

Based on this consideration, a more general approach using the best polynomial approximation is introduced in the follow sections. In the presented approach, the frequency points corresponding to the Chebyshev nodes are found by transformation of coordinates, and MOM is used to compute the electric currents at these points, then the electric currents at any point within the given frequency band can be obtained from the Chebyshev polynomial functions[4 - 6].

In the following sections, the theory of Chebyshev approximation is discussed, and numerical results are presented for a PEC sphere and other multiple PEC objects. Compared with the results obtained by direct solution, the presented approach is found to be superior in terms of the CPU time without sacrificing much memory.

II. THEORY AND FORMULATIONS

A. Electric Field Equation and MOM

Consider the problem of electromagnetic scattering of arbitrarily shaped three- dimensional body. Let S denotes the surface of the body. The boundary condition requires the tangential component of the total electric field to vanish on the surface of the PEC body,

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$$(\mathbf{E}^{inc}+\mathbf{E}^{scat})_{\tan}=0 \tag{1}$$

where $\mathbf{E}^{inc}$ and $\mathbf{E}^{scat}$ denote incident and scattered electric field, respectively.

By MOM method with the Rao-Wilton- Glisson (RWG) basis functions introduced in [7 - 9], equation (1) will be reduced to a matrix equation

$$\mathbf{Z}(k)\mathbf{I}(k)=\mathbf{V}(k) \tag{2}$$

where k is the wavenumber, $\mathbf{Z}$ is a $N\times N$ impedance matrix containing information about the electromagnetic interaction between the basis functions, $\mathbf{V}$ is a vector of size N containing information about the incident field and $\mathbf{I}$ is a vector of size N that denotes the unknown coefficients in RWG basis functions.

B. Theory of Chebyshev Approximation

The electric current in equation (2) is calculated at a single frequency point. If one needs the RCS over a broad frequency band, this calculation must be repeated for different frequency points within interest band, which must be time consuming. Thus the Chebyshev approximation theory is introduced to complete the scattering analysis over a broad frequency band [10], [11].

For $H_n=span\{1,x,x^2,\cdots x^n\}$, $P_n(x)\in H_n$, $f(x)\in C[a,b]$, where $[a,b]$ is the range of investigation. The L^∞ error is defined as

$$\Delta(f,P_n)=\left\|f-p_n\right\|_\infty=\max_{a\le x\le b}\left|f(x)-P_n(x)\right| \tag{3}$$

and the infimum of $\Delta(f,P_n)$ is

$$E_n=\inf_{P_n\in H_n}\{\Delta(f,P_n)\}=\inf_{P_n\in H_n}\max_{a\le x\le b}\left|f(x)-P_n(x)\right|. \tag{4}$$

For $f(x)\in C[a,b]$, if there is a $P_n^*(x)\ \in H_n$ and $\Delta(f,P_n^*)=E_n$, then $P_n^*(x)$ is the best polynomial approximations in H_n for $f(x)$.

Theorem 1: For a given $f(x)\in C[a,b]$, there always exists a unique $P_n^*(x)\in\ H_n(x)$, such that

$$\left\|f(x)-P_n^*(x)\right\|_\infty=E_n. \tag{5}$$

Theorem 2: Let $f(x)\in C[a,b]$ be given, if there are $n+2$ points $a\le x_1\le\ x_2\le\cdots\ x_{n+2}\le b$ that satisfy

$$P(x_l)-f(x_l)=(-1)^l\sigma\left\|P(x)-f(x)\right\|_\infty \tag{6}$$

where $\sigma=\pm 1$, then $P(x)$ will be the best polynomial approximations in H_n for $f(x)$.

The Chebyshev polynomial of degree n is denoted $T_n(x)$, which is given by the explicit formula

$$T_0(x)=1,T_1(x)=x$$
$$T_{n+1}(x)=2xT_n(x)-T_{n-1}(x)\quad n\ge 1. \tag{7}$$

While the Chebyshev approximation formula for $f(x)\in C[-1,1]$ is given by

$$f(x)=-\frac{1}{2}c_0+\sum_{l=0}^{\infty}c_lT_l(x)=P_n(x)+\sum_{l=n+1}^{\infty}c_lT_l(x) \tag{8}$$

where $x\in[-1,1]$

hence $f(x)-P_n(x)\approx c_{n+1}T_{n+1}(x)$.

Since for $T_{n+1}(x)$ there are $n+2$ points

$$x_l=\cos(\frac{l\pi}{n+1})\quad (l=0,1,\cdots n+1)$$

which satisfies equation (6), the Chebyshev series would be the best polynomial approximations in $C[-1,1]$ by theorem 2.

C. Specified Formulations for Frequency Sweep Scattering Analysis

To introduce the theory into scattering analysis and illuminate the approach in detail, the specified formulations for frequency sweep analysis are presented in this section.

For a given frequency range $f\in[f_a,f_b]$, and the wavenumber $k\in[k_a,k_b]$, the transformation of coordinates is used as

$$\tilde{k}=\frac{2k-(k_a+k_b)}{k_b-k_a}, \tag{9}$$

such that $\tilde{k}\in[-1,1]$.

Then the electric current $I(k)$ will be calculated using

$$\mathbf{I}(k)=\mathbf{I}(\frac{\tilde{k}(k_b-k_a)+(k_b+k_a)}{2}) \tag{10}$$

and the Chebyshev approximation for $\mathbf{I}(k)$ is given by

$$\mathbf{I}(k)=\mathbf{I}(\frac{\tilde{k}(k_b-k_a)+(k_b+k_a)}{2})\approx\sum_{l=0}^{n}\mathbf{c}_lT_l(\tilde{k})-\frac{\mathbf{c}_0}{2}, \tag{11}$$

having

$$\mathbf{c}_l = \frac{2}{n+1}\sum_{i=0}^{n+1}\mathbf{I}(k_i)T_l(\tilde{k}_i) \quad (12)$$

where $\tilde{k}_i\ (i = 0,1,2,\cdots,n)$ are the Chebyshev nodes for $T_n(\tilde{k})$, and $k_i \in [k_a, k_b]$ can be obtained using

$$k_i = \frac{\tilde{k}_i(k_b - k_a) + (k_a + k_b)}{2} \quad (13)$$

To sum up, to get the Chebyshev approximation, the $n+1$ Chebyshev nodes for $T_n(\tilde{k})$ ($\tilde{k} \in [-1,1]$) must be computed firstly, then the wavenumbers k_i should be calculated using equation (13) and the surface electric current $\mathbf{I}(k_i)$ can be obtained by method of moments. Substituting $\mathbf{I}(k_i)$ into equation (12), one can obtain the Chebyshev series coefficients $\mathbf{c}_l$, and the surface electric current over the wide band is calculated using equation (11). Then the RCS over the given band is obtained.

III. NUMERICAL RESULTS

The Chebyshev approximation scheme described above has been applied to a sphere, two cubes, and three spheres, which are illuminated by a plane wave $\mathbf{E}^{inc} = \mathbf{e_x}e^{-jk_z z}$.

In Fig. 1, the radius of the sphere is chosen to be 0.3 cm, and the frequency band calculated is from 10 GHz to 55 GHz. The surface is discretized into 500 triangular elements. The CPU time for the direct solution of all output points is 4898.11 seconds, and that for AWE technique is 774.69 seconds, while the CPU time for the presented approximation approach is 756.43 seconds. The result is compared with that of AWE technique, and that of the direct solution of EFIE.

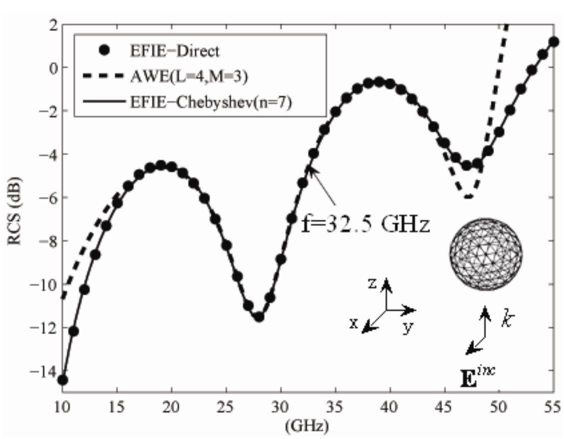

Fig. 1. RCS of a sphere from 10 GHz to 55 GHz.

It is found that the result obtained by Chebyshev method agree to that provided by direct solution very well, while the AWE technique does not. Where the order of the Chebyshev series is chosen to be 7, which is equal to the order of Taylor series. In figure 1, L and M denotes the order of the numerator and denominator in padé approximation.

As a second example, the RCS of two PEC cubes is considered, the length of each cube is chosen to be 0.5 cm, and the distance between them is 1 cm. The RCS is calculated from 5 GHz to 35 GHz, as shown in Fig. 2. The surface of the two cubes is discretized into 864 triangular elements resulting in 1296 RWG functions. The CPU times required for the direct solution and the approximation approach (n=7) are 6.6987 hours and 46.4 minutes, respectively.

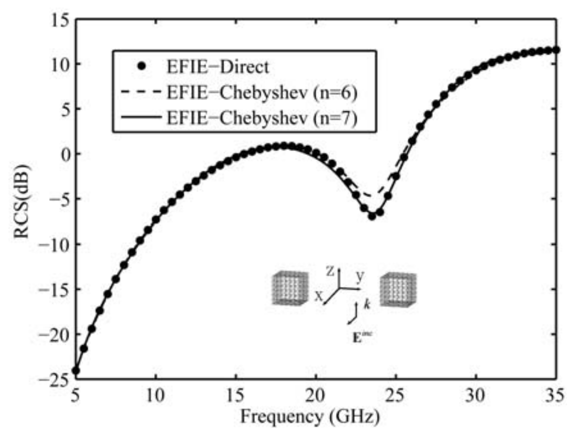

Fig. 2. RCS of two cubes from 5 GHz to 35 GHz.

Figure 3, represents the configuration of three PEC spheres that are uniformly placed on the y-axis. The radius of each sphere is 0.3 cm, and the center distance between any two neighbor spheres is 1 cm. The RCS starting from 16 GHz to 36 GHz is calculated and presented in Fig. 4. The CPU times for the direct solution and the approximation approach (n=8) are 5.5905 hours and 45.2 minutes, respectively.

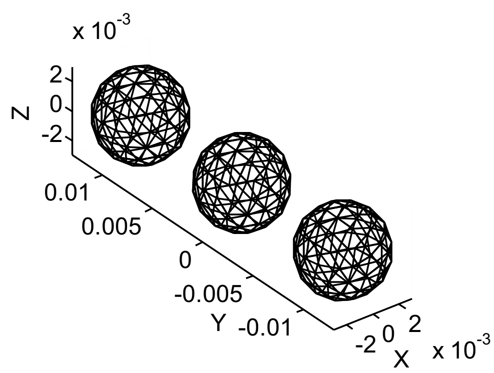

Fig. 3. Three PEC spheres placed along the y-axis.

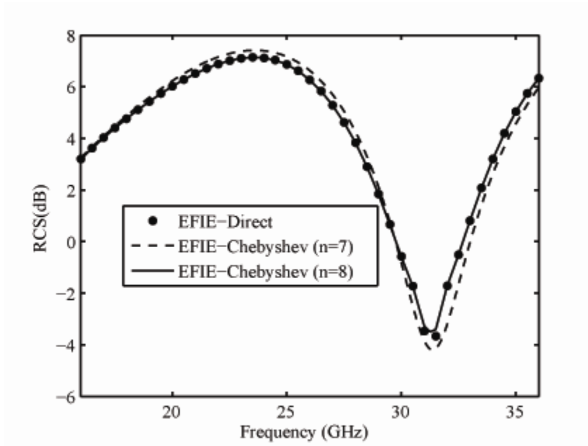

Fig. 4. RCS of three spheres from 16 GHz to 36 GHz.

It can be clearly observed from the data presented in Figs. 2 and 4, that the results obtained by the method presented vary with the value of n, and the accuracy is improved as the order increase.

All the computations reported are achieved by Matlab 7.0 on a PIV2.66G personal computer.

IV. CONCLUSION

The best polynomial approximation, implemented by Chebyshev approximation, for frequency sweep scattering analysis is presented. The RCS for different PEC objects are computed and compared with the direct solution by MOM at each of the frequency points, the presented approach is found to be superior in terms of the CPU time without sacrificing much memory. The accuracy of the presented approximation approach and its relation to the order of the best polynomial series are topics of interest for future research. With these topics addressed, the method presented will be of good application in obtaining the RCS over a desired frequency band using a frequency-domain technique.

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