Fast and accurate radar cross-section computation over a broad frequency band using the best uniform rational approximation

M.S. Chen, X.L. Wu, W. Sha and Z.X. Huang

Abstract: The method of moments (MOM) in conjugation with the best uniform rational approximation is applied to predict the broad-band radar cross-section of arbitrarily shaped three-dimensional (3D) bodies. The surface integral equations are solved using MOM to obtain the equivalent surface currents at the frequency points corresponding to the Chebyshev nodes, and then the surface current within the given frequency band is represented by the Chebyshev series. To improve the accuracy, the Chebyshev series is matched via the Maehly approximation to a rational function, which can be as good as the best uniform rational approximation. Using the rational function, the surface current is obtained at any frequency point within the frequency band. Numerical results for 3D arbitrarily shaped perfectly electric conducting objects and homogenous dielectric bodies are considered. Compared with the asymptotic waveform evaluation technique and the model-based parameter estimation, the proposed technique is accurate in much broader frequency band with lower memory required.

1 Introduction

The solution of surface integral equation (SIE) using the method of moments (MOM) has been a very useful tool for accurately predicting the radar cross-section (RCS) of arbitrarily shaped three-dimensional (3D) objects in the frequency domain [1]. For a 3D object, its surface is divided into sub-domains, and the integral equation is then reduced to be a matrix equation which is solved for the equivalent surface currents. The MOM is advantageous because it limits the unknown current on the surface of an object and it satisfies the radiation condition via Green's function. However, the method results in a dense matrix that is computationally expensive to generate, and the solution of the algebraic equations becomes the major time-consuming operation in MOM [2].

To obtain the RCS over a broad-band using MOM, one has to repeat the calculation at each frequency point over the band of interest. If the RCS is highly frequency-dependent, one needs to do the calculations at finer increment of frequency to obtain an accurate representation of the frequency response, which must be computationally intensive.

Over the past few years, there is a strong desire to find approximate solution techniques that can efficiently simulate the frequency response over a broad frequency band. Among the earlier attempts to achieve a fast frequency sweep analysis by MOM and other frequency-domain techniques, the asymptotic waveform evaluation (AWE) technique and the model-based parameter estimation (MBPE) are two of the most prevalent methods, which have been successfully used in various electromagnetic problems [3–9]. In AWE technique, Taylor series expansion is generated for a specific value of the system parameter (frequency, angle, distance etc.), and the rational function approach obtained from padé approximation is used to improve the accuracy of the numerical solution. It was shown that the application of AWE requires less CPU time to obtain a frequency response and it can speed up the analysis by an order of magnitude. However, the accuracy of the Taylor series is limited by the radius of convergence, and the high derivatives of the dense impedance matrix must be stored to compute the coefficients, which will greatly increase the memory consumed. The MBPE method is similar to the AWE but more flexible, in which the surface current is expanded directly as a rational function. The coefficients of the rational function in MBPE are obtained using the frequency data and the frequency-derivative data, and the memory consumed is reduced when compared with AWE when the data are sampled from more than one frequency point.

In this paper, a new fast frequency-sweep technique was proposed, which applies the Maehly approximation technique to the MOM solution of the SIE for different perfectly electric conducting (PEC) objects or homogenous dielectric objects. For a given frequency band, the frequency points corresponding to the Chebyshev nodes are found by transformation of coordinates, and the MOM is used to compute the currents at these points, then the surface current is represented by the Chebyshev series which can be used as good as the best polynomial approximation [10]. Akin to the padé approximation, the coefficients of the Chebyshev series are then matched via the Maehly approximation to a rational function to improve the accuracy and obtain acceptable solution in a much broader frequency band, and the rational function obtained can be used as the best uniform rational approximation [11, 12].

The rest of the paper is organised as follows. In Section 2, the theory of the best uniform approximation is presented.

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Numerical results for PEC sphere, 3D multiple objects and homogenous dielectric bodies are considered in Section 3, followed by the discussion on the choice of the number of Chebyshev nodes presented in Section 4. Finally, conclusion is outlined in Section 5.

2 Theory and formulations

Using MOM with the Rao–Wilton–Glisson (RWG) basis functions [13], the surface equations mentioned above can be reduced to a matrix equation

$$\boldsymbol{Z}(k)\boldsymbol{I}(k) = \boldsymbol{V}(k) \tag{1}$$

where $\boldsymbol{Z}$ is a known $N \times N$ impedance matrix, $\boldsymbol{V}$ a known vector of size N containing information about the incident field and $\boldsymbol{I}$ a vector of size N containing the unknown basis coefficients.

The current vector $\boldsymbol{I}$ is calculated at a single frequency point. If one needs the RCS over a broad frequency band, this calculation must be repeated for different frequency points within the interest band, which must be time-consuming. Thus, the best uniform rational approximation theory is introduced to complete the scattering analysis over a broad frequency band.

For a given frequency range $f \in [f_a, f_b]$ and the wave-number $k \in [k_a, k_b]$, the transformation of coordinates is used as

$$\tilde{k} = \frac{2k - (k_a + k_b)}{k_b - k_a} \tag{2}$$

such that $\tilde{k} \in [-1, 1]$.

Then the electric current $\boldsymbol{I}(k)$ will be calculated by

$$\boldsymbol{I}(k) = \boldsymbol{I}\left(\frac{\tilde{k}(k_b - k_a) + (k_b + k_a)}{2}\right) \tag{3}$$

and the Chebyshev approximation for $\boldsymbol{I}(k)$ is given by

$$\boldsymbol{I}(k) = \boldsymbol{I}\left(\frac{\tilde{k}(k_b - k_a) + (k_b + k_a)}{2}\right) \simeq \sum_{l=0}^{n} c_l T_l(\tilde{k}) - \frac{c_0}{2} \tag{4}$$

$$c_l = \frac{2}{n+1}\sum_{j=0}^{n+1} \boldsymbol{I}(k_j)T_l(\tilde{k}_j) \tag{5}$$

$$T_0(x) = 1, \quad T_1(x) = x, \ldots, T_{l+1}(x) = 2xT_l(x) - T_{l-1}(x) \tag{6}$$

where $\tilde{k}_j$ $(j = 0, 1, 2, \ldots, n)$ are the Chebyshev nodes for $T_{n+1}(\tilde{k})$

$$\tilde{k}_j = \cos\left(\frac{j + (1/2)}{n+1}\pi\right) \quad (j = 0, 1, \ldots, n) \tag{7}$$

and $k_j \in [k_a, k_b]$ can be obtained by

$$k_j = \frac{\tilde{k}_j(k_b - k_a) + (k_a + k_b)}{2} \tag{8}$$

To improve the accuracy of the numerical solution, the Chebyshev series is replaced by a rational function named Maehly approximation

$$\begin{aligned}\boldsymbol{I}(k) \simeq R_{LM}(\tilde{k}) &= \frac{P_L(\tilde{k})}{Q_M(\tilde{k})} \\ &= \frac{a_0T_0(\tilde{k}) + a_1T_1(\tilde{k}) + \cdots a_LT_L(\tilde{k})}{b_0T_0(\tilde{k}) + b_1T_1(\tilde{k}) + \cdots b_MT_M(\tilde{k})}\end{aligned} \tag{9}$$

where b_0 is set to be 1 as the rational function can be divided by an arbitrary constant.

Substituting (9) into (4) and using the identity

$$T_p(x)T_q(x) = \frac{1}{2}\left(T_{p+q}(x) + T_{|p-q|}(x)\right) \tag{10}$$

the unknown coefficients a_i $(i = 0, 1, \ldots, L)$ and b_j $(j = 1, 2, \ldots, M)$ can be solved by

$$\begin{cases} a_0 = \frac{1}{2}b_0c_0 + \frac{1}{2}\sum_{j=1}^{M} b_jc_j \\ a_i = c_i + \frac{1}{4}b_ic_0 + \frac{1}{2}\sum_{j=1}^{M} b_j\left(c_{j+i} + c_{|j-i|}\right) \quad i = 1, 2, \ldots, L \end{cases} \tag{11}$$

$$\begin{pmatrix} c_{L+2}+c_L & c_{L+3}+c_{L-1} & \cdots & c_{L+M+1}+c_{L-M+1} \\ c_{L+3}+c_{L+1} & c_{L+4}+c_L & \cdots & c_{L+M+2}+c_{L-M+2} \\ \vdots & \vdots & & \vdots \\ c_{L+M+1}+c_{L+M-1} & c_{L+M+2}+c_{L+M-2} & \cdots & c_{L+2M}+c_L \end{pmatrix}\begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_M \end{pmatrix} = -2\begin{pmatrix} c_{L+1} \\ c_{L+2} \\ \vdots \\ c_{L+M} \end{pmatrix} \tag{12}$$

The error function is given by

$$E_n = R_{LM}(\tilde{k}) - \boldsymbol{I}(k) = \frac{1}{Q_M(\tilde{k})}\sum_{p=L+M+1}^{\infty} h_pT_p(\tilde{k}) \tag{13}$$

where the coefficient h_p is defined by

$$P_M(\tilde{k}) - Q_L(\tilde{k})\boldsymbol{I}(k) = \sum_{p=L+M+1}^{\infty} h_pT_p(\tilde{k}) \tag{14}$$

Since $b_0 = 1$, and h_p attenuates quickly, the error function can be approximated by

$$E_n \simeq h_{L+M+1}T_{L+M+1}(\tilde{k}) \tag{15}$$

$$h_{L+M+1} = c_{L+M+1} + \frac{1}{2}\sum_{i=1}^{M} b_i(c_{L+M+i+1} + c_{L+M-i+1}) \tag{16}$$

Hence $R_{LM}(\tilde{k})$ can be used as the best uniform rational approximation for $\boldsymbol{I}(k)$.

3 Numerical results

3.1 Numerical examples for PEC objects

As the first example, a PEC sphere with a radius of 0.3 cm is considered, and the surface of the sphere is discretised into 720 triangular elements. As shown in Fig. 1, the results obtained by the best uniform polynomial approximation performed by Chebyshev series with order of 7 agrees well with that of the direct solution for electric field integral

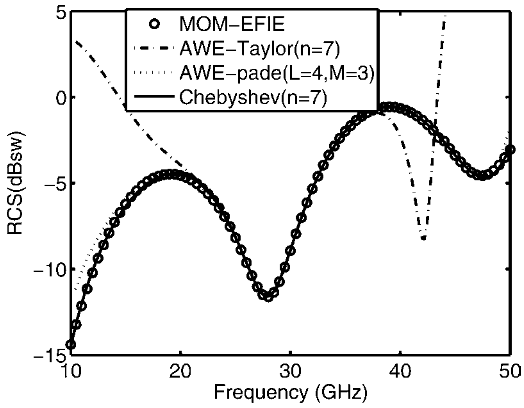

Fig. 1 *RCS frequency response of a PEC sphere with a radius of 0.3 cm*

equation (EFIE). When the AWE scheme is applied with its expansion point at 30 GHz, the Taylor series with the same order produces an accurate result only in a narrow frequency band ranging from 22 to 38 GHz, and the padé approximation obtains the accurate solution from 15 to 48 GHz. The CPU time required for direct solution, AWE and Chebyshev are 980, 89 and 92 s, respectively. It is shown that, even before transformed to be rational approximation, the Chebyshev series produces more accurate results than the AWE- padé scheme.

A target that is made up by two cubes illuminated by a pane wave propagating in the z-direction and E-polarised in the x-direction is considered. The two cubes are uniformly placed on y-axis, the length of the cube is chosen to be 1 cm, and the centre distance of them is 5 cm. As shown in Fig. 2, the accuracy of AWE (with expanded point at 18.5 GHz) will be severely limited by the radius of convergence in Taylor series, and the MBPE method which is applied with two sampled frequency points at 13 and 24 GHz gives a better result, whereas the Maehly method produces more accurate results over the entire frequency band. The surface of the two cubes is discretised into 1536 triangular elements. With a frequency step of 1 GHz, it takes the direct solution of EFIE 2494 s to obtain the solution from 2 to 35 GHz. With 0.1 GHz increments over the entire band, the AWE scheme, MBPE method and Maehly method consumed 589, 592 and 596 s, respectively.

To consider the application of the proposed method to the solution of combined field integral equation [14], a PEC sphere with radius of 0.5 cm is considered. As we can see from Fig. 3, the Maehly method produces accurate result from 5 to 45 GHz, whereas the AWE scheme and MBPE method obtain a more narrow solution. The surface of the object is discretised into 980 triangular elements.

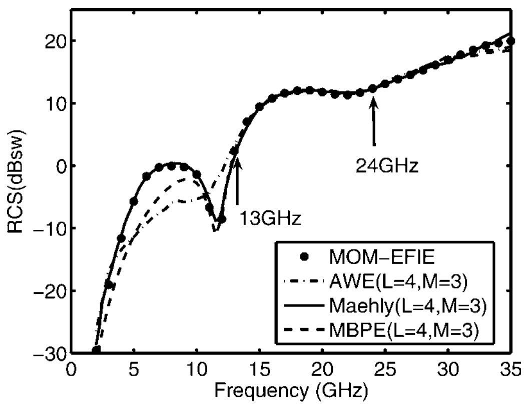

Fig. 2 *RCS frequency response of two cubes*

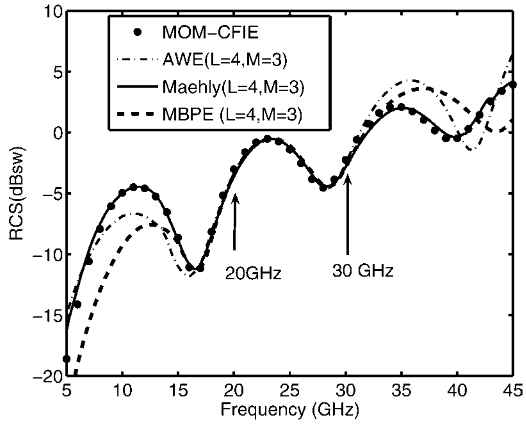

Fig. 3 *RCS frequency response of a PEC sphere with a radius of 0.5 cm*

3.2 Numerical examples for homogenous dielectric objects

When considering the scattering problem of an arbitrarily shaped homogeneous dielectric body characterised by permittivity ε_2 and permeability μ_2 and immersed in an infinite and homogeneous medium having permittivity ε_1 and permeability μ_1, the so-called PMCHW equation [15], named after Poggio, Miller, Chang, Harrington, and Wu, who originally developed the formulation, is usually applied.

For a dispersive dielectric object, ε_2 and μ_2 can be a function of frequency. For simplicity, the situation that only ε_2 varies with frequency is considered. The complex permittivity of a homogenous dielectric can be described by the Debye model.

$$\varepsilon_2(\omega) = \varepsilon'_{2\infty} + \frac{\varepsilon'_{2s} - \varepsilon'_{2\infty}}{1 + j\omega\tau_e} \tag{17}$$

where ε'_{2s} denotes the static dielectric constant, $\varepsilon'_{2\infty}$ the optical dielectric constant, and τ_e a relaxation time constant related to the original relaxation time constant τ by

$$\tau_e = \tau\frac{\varepsilon'_{2s} + 2\varepsilon_0}{\varepsilon'_{2\infty} + 2\varepsilon_0} \tag{18}$$

in which ε_0 denotes the permittivity of free space.

To compare the accuracy of the Chebyshev approximation with the Maehly approximation, a homogenous

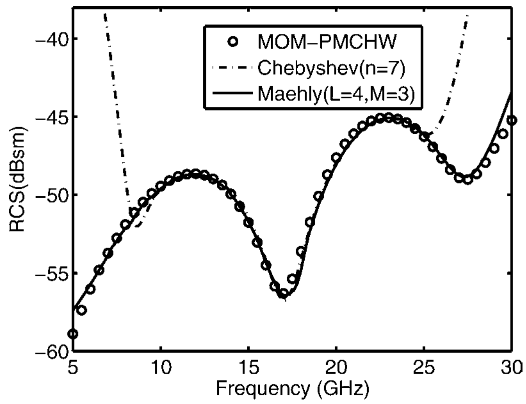

Fig. 4 *RCS frequency response of a homogenous dielectric sphere*

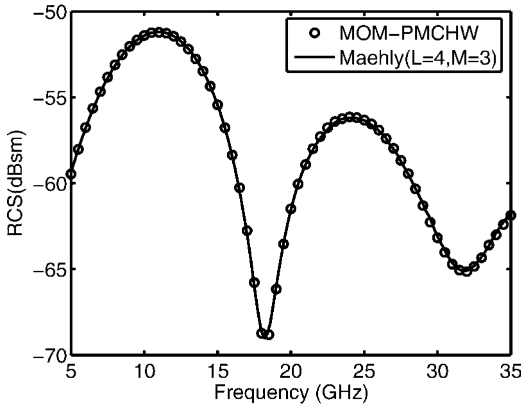

Fig. 5 *RCS frequency response of a dispersive dielectric sphere*

sphere with radius of 0.5 cm immersed in free space is considered. The PMCHW equation is built to obtain the solution. The surface of the sphere is modelled by 720 triangular patches, resulting in 1080 edges and thus 2160 unknowns. The relative permittivity of the sphere is $\varepsilon_r = 2.0$ and relative permeability is set to be $\mu_r = 1.0$. As can be seen from Fig. 4, the accuracy of the Chebyshev series is improved greatly after transformed to be a rational function via Maehly approximation. In Fig. 5, a dispersive dielectric sphere of the same size is considered, the permittivity of the object is characterised by $\varepsilon'_{rs} = 2.0$, $\varepsilon'_{r\infty} = 1.0$ and $\tau = 7.59$ ps/rad. As a result, as frequency varies from 5 to 35 GHz, the real part of the relative permittivity varies from 1.91 to 1.17 and the imaginary part varies from 0.29 to 0.37. As shown in Fig. 5, the Maehly method produces accurate result for the dispersive dielectric sphere in the given frequency band, and it takes 4800 s for the direct solution method to obtain the backscatter RCS, whereas Maehly method consumed only 633 s.

As the final example, a dielectric cube having a side length of 1 cm is considered. The surface of the object is modeled by 644 triangular elements. The permittivity of the object is characterised by $\varepsilon'_{rs} = 2.0$, $\varepsilon'_{r\infty} = 1.0$ and $\tau = 3.2$ ps/rad. With a frequency step of 0.5 GHz, it takes the direct method 2347 s to complete the calculation from 5 to 25 GHz. In contrast, the Maehly method produces the solution with 0.1 GHz increments in 461 s. Meanwhile, a non-dispersive cube of the same size and having a relative permittivity 2.0 is also shown in Fig. 6.

All the computations reported above have been carried out on a Personal Computer with Pentium IV-2.66 GHz processor and 1 GB RAM. The code has been written in Microsoft Visual C++ 6.0.

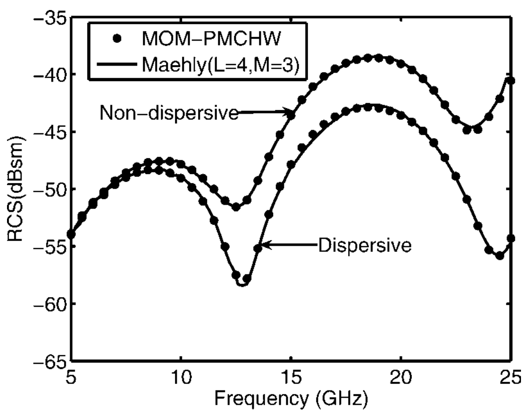

Fig. 6 *RCS frequency response of a dielectric cube with a side length of 1 cm*

4 Discussion

As we know, for the application of the presented method to a specific problem (the object and the frequency band are given), how to choose the number of Chebyshev nodes to cover the entire band must be an important issue. Apparently, one cannot start from a small number and gradually increase since the previously computed data cannot be reused. Therefore the n in (4) should be estimated in advance. After numerical simulation, an experienced formula is obtained here to estimate n under the condition that the relative RCS error is acceptable

$$n = \frac{\alpha}{2\pi}\Delta k\tilde{d} \tag{19}$$

where Δk is the increment of the wavenumber in the interior region and $\tilde{d}$ stands for the equivalent source-to-field distance, and it is set to be the maximum of distance from source to field in this paper for the sake of safety. The parameter α is a constant range from 7 to 9, and the value of it increases as the shape of the object becomes complex.

The relative root mean square (RMS) RCS error of a specific object is calculated by

$$\mathrm{Err}_{\mathrm{RMS}} = \left\{\frac{1}{N}\sum_{j=1}^{N}\left|10\log 10\left(\frac{\hat{\sigma}(f_j)}{\sigma(f_j)}\right)\right|^2\right\}^{1/2} \tag{20}$$

where f_j ($j = 1, 2, \ldots, N$) represent N selected frequency points, while $\hat{\sigma}(f_j)$ and $\sigma(f_j)$ are the RCS obtained by the proposed method and the direct solution method, respectively.

To validate the above analysis, a PEC sphere is considered in Fig. 7, and the relative RMS RCS error varying with the value of n is calculated for different $\Delta k\tilde{d}$. When one set α to be 7, the relative RMS RCS error is always lower than 0.4 dB, whereas the relative RMS RCS errors described by (20) for Figs. 2 and 3 are 0.5423 and 0.4712 dB, respectively. As shown in Fig. 8, the relative RMS error of a PEC cube is analysed. The error will be acceptable if n is computed by (19), and the accuracy is improved as n increased.

In Fig. 9, a homogenous dielectric sphere is considered, the relative permittivity of the sphere is $\varepsilon_r = 2.0$ and

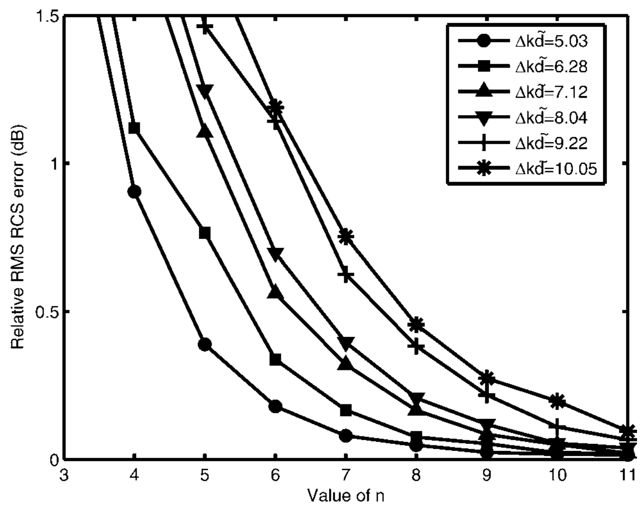

Fig. 7 *Relative RMS RCS error of PEC sphere varies with the value of n*

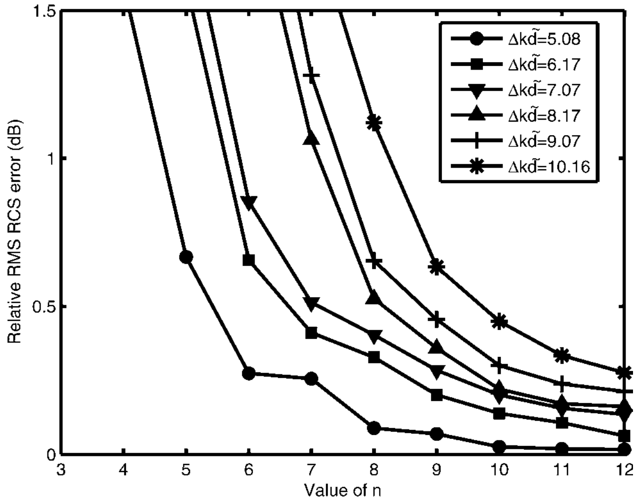

Fig. 8 *Relative RMS RCS error of PEC cube varies with the value of n*

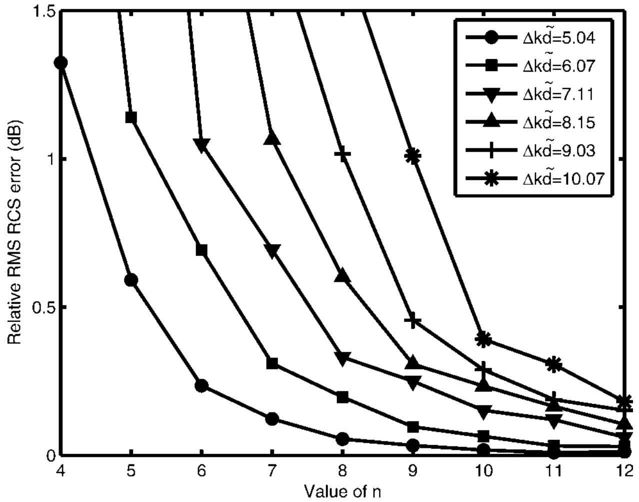

Fig. 9 *Relative RMS RCS error of dielectric sphere varies with the value of n*

relative permeability is set to be $\mu_r = 1.0$. As we can see from Fig. 9, the experienced formula can also be applied to the analysis of homogenous dielectric object.

When multiple objects are considered and if the centre distance is so far that the interaction between them can be ignored, they should be treated as isolated objects when the experienced formula is applied, and the maximum of n should be adopted.

5 Conclusion

An implementation of the best uniform rational approximation for MOM solution over a broad frequency band is proposed. The RCS of different PEC or dielectric objects are computed and compared with the direct solution method, MBPE method and AWE technique. Compared with the direct solution by MOM, the presented method is found to be superior in terms of the CPU time to obtain RCS frequency response. Compared with the MBPE method and AWE scheme, the presented technique can obtain accurate results over much broader frequency band without increasing any memory, and it is shown that the use of Maehly method can speed up the calculations as much as AWE or MBPE.

The accuracy of the proposed approximation approach and its relation to the order of the best polynomial series is preliminary discussed, and further research on it, especially for multiple objects, can be the topic of interest for future work.

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