Fast and accurate radar cross-section computation over a broad frequency band using the best uniform rational approximation

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Abstract: The method of moments (MOM) in conjunction with the best uniform rational approximation is applied to predict the broad-band radar cross-section of arbitrarily shaped three-dimensional (3D) bodies. The surface integral equations are solved using MOM to obtain the equivalent surface currents at the frequency points corresponding to the Chebyshev nodes, and then the surface current within the frequency band is represented by the Chebyshev series. To improve the accuracy, the Chebyshev series is matched via the Maehly approximation to a rational function, which can be as good as the best uniform rational approximation. Using the rational function, the surface current is obtained at any frequency point within the frequency band. Numerical results for 3D arbitrarily shaped perfectly electric conducting objects and homogenous dielectric bodies are considered. Compared with the asymptotic waveform evaluation technique and the model-based parameter estimation, the proposed technique is accurate in much broader frequency band with lower memory required.

1 Introduction

The solution of surface integral equation (SIE) using the method of moments (MOM) has been a very useful tool for accurately predicting the radar cross-section (RCS) of arbitrarily shaped three-dimensional (3D) objects in the frequency domain [1]. For a 3D object, its surface is divided into sub-domains, and the integral equation is then reduced to be a matrix equation which is solved for the equivalent surface currents. The MOM is advantageous because it limits the unknown current on the surface of an object and it satisfies the radiation condition via Green’s function. However, the method results in a dense matrix that is computationally expensive to generate, and the solution of the algebraic equations becomes the major time-consuming operation in MOM [2].

To obtain the RCS over a broad-band using MOM, one has to repeat the calculation at each frequency point over the band of interest. If the RCS is highly frequency-dependent, one needs to do the calculations at finer increment of frequency to obtain an accurate representation of the frequency response, which must be computationally intensive.

Over the past few years, there is a strong desire to find approximate solution techniques that can efficiently simulate the frequency response over a broad frequency band. Among the earlier attempts to achieve a fast frequency sweep analysis by MOM and other frequency-domain techniques, the asymptotic waveform evaluation (AWE) technique and the model-based parameter estimation (MBPE) are two of the most prevalent methods, which have been successfully used in various electromagnetic problems [3–9]. In AWE technique, Taylor series expansion is generated for a specific value of the system parameter (frequency, angle, distance etc.), and the rational function approach obtained from padé approximation is used to improve the accuracy of the numerical solution. It was shown that the application of AWE requires less CPU time to obtain a frequency response and it can speed up the analysis by an order of magnitude. However, the accuracy of the Taylor series is limited by the radius of convergence, and the high derivatives of the dense impedance matrix must be stored to compute the coefficients, which will greatly increase the memory consumed. The MBPE method is similar to the AWE but more flexible, in which the surface current is expanded directly as a rational function. The coefficients of the rational function in MBPE are obtained using the frequency data and the frequency-derivative data, and the memory consumed is reduced when compared with AWE when the data are sampled from more than one frequency point.

In this paper, a new fast frequency-sweep technique was proposed, which applies the Maehly approximation technique to the MOM solution of the SIE for different perfectly electric conducting (PEC) objects or homogenous dielectric objects. For a given frequency band, the frequency points corresponding to the Chebyshev nodes are found by transformation of coordinates, and the MOM is used to compute the currents at these points, then the surface current is represented by the Chebyshev series which can be used as good as the best polynomial approximation [10]. Akin to the padé approximation, the coefficients of the Chebyshev series are then matched via the Maehly approximation to a rational function to improve the accuracy and obtain acceptable solution in a much broader frequency band, and the rational function obtained can be used as the best uniform rational approximation [11, 12].

The rest of the paper is organised as follows. In Section 2, the theory of the best uniform approximation is presented.

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Numerical results for PEC sphere, 3D multiple objects and homogeneous dielectric bodies are considered in Section 3, followed by the discussion on the choice of the number of Chebyshev nodes presented in Section 4. Finally, conclusion is outlined in Section 5.

2 Theory and formulations

Using MOM with the Rao–Wilton–Glisson (RWG) basis functions [13], the surface equations mentioned above can be reduced to a matrix equation

\[ Z(k)I(k) = V(k) \]  

(1)

where Z is a known \( N \times N \) impedance matrix, V a known vector of size N containing information about the incident field and I a vector of size N containing the unknown basis coefficients.

The current vector I is calculated at a single frequency point. If one needs the RCS over a broad frequency band, this calculation must be repeated for different frequency points within the interest band, which must be time-consuming. Thus, the best uniform rational approximation theory is introduced to complete the scattering analysis over a broad frequency band.

For a given frequency range \( f \in [f_a, f_b] \) and the wavenumber \( k \in [k_a, k_b] \), the transformation of coordinates is used as

\[ \tilde{k} = \frac{2k - (k_a + k_b)}{k_b - k_a} \]  

(2)

such that \( \tilde{k} \in [-1, 1] \).

Then the electric current \( I(k) \) will be calculated by

\[ I(k) = I \left( \frac{\tilde{k}(k_b - k_a) + (k_a + k_b)}{2} \right) \]  

(3)

and the Chebyshev approximation for \( I(k) \) is given by

\[ I(k) \approx \sum_{j=0}^{n} c_j T_j(\tilde{k}) - c_0 \]  

(4)

\[ c_j = \frac{2}{n+1} \sum_{j=0}^{n+1} I(k_j) T_j(\tilde{k}) \]  

(5)

\[ T_0(x) = 1, \quad T_1(x) = x, \quad \ldots, \]  

(6)

where \( \tilde{k}_j (j = 0, 1, 2, \ldots, n) \) are the Chebyshev nodes for \( T_{n+1}(\tilde{k}) \)

\[ \tilde{k}_j = \cos \left( \frac{j + (1/2)}{n + 1} \pi \right) \quad (j = 0, 1, \ldots, n) \]  

(7)

and \( k_j \in [k_a, k_b] \) can be obtained by

\[ k_j = \frac{\tilde{k}_j(k_b - k_a) + (k_a + k_b)}{2} \]  

(8)

To improve the accuracy of the numerical solution, the Chebyshev series is replaced by a rational function named Maehly approximation

\[ I(k) \approx R_{LM}(\tilde{k}) = \frac{P_{LM}(\tilde{k})}{Q_{LM}(\tilde{k})} \]  

(9)

where \( b_0 \) is set to be 1 as the rational function can be divided by an arbitrary constant.

Substituting (9) into (4) and using the identity

\[ T_p(x)T_q(x) = \frac{1}{2} \left( T_{p+q}(x) + T_{|p-q|}(x) \right) \]  

(10)

the unknown coefficients \( a_i, (i = 0, 1, \ldots, L) \) and \( b_j (j = 1, 2, \ldots, M) \) can be solved by

\[
\begin{align*}
0 &= \frac{1}{2} b_0 c_0 + \frac{1}{2} \sum_{j=1}^{M} b_j c_j \\
\frac{1}{2} b_0 c_0 + \frac{1}{2} \sum_{j=1}^{M} b_j c_j &= \sum_{i=1}^{L} \left[ c_{i+1} + c_{i-1} \right] \\
\frac{1}{2} b_0 c_0 + \frac{1}{2} \sum_{j=1}^{M} b_j c_j &= 2 \left[ c_1 \right] \\
\frac{1}{2} b_0 c_0 + \frac{1}{2} \sum_{j=1}^{M} b_j c_j &= \cdots \\
\frac{1}{2} b_0 c_0 + \frac{1}{2} \sum_{j=1}^{M} b_j c_j &= c_{L+1} \\
\frac{1}{2} b_0 c_0 + \frac{1}{2} \sum_{j=1}^{M} b_j c_j &= c_{L+2} \\
\frac{1}{2} b_0 c_0 + \frac{1}{2} \sum_{j=1}^{M} b_j c_j &= \cdots \\
\frac{1}{2} b_0 c_0 + \frac{1}{2} \sum_{j=1}^{M} b_j c_j &= c_{L+M} \\
\end{align*}
\]

(11)

The error function is given by

\[ E_n = R_{LM}(\tilde{k}) - I(k) = \frac{1}{Q_{LM}(\tilde{k})} \sum_{p=L+M+1}^{\infty} h_p T_p(\tilde{k}) \]  

(13)

where the coefficient \( h_p \) is defined by

\[ P_{LM}(\tilde{k}) - Q_{LM}(\tilde{k}) I(k) = \sum_{p=L+M+1}^{\infty} h_p T_p(\tilde{k}) \]  

(14)

Since \( b_0 = 1 \), and \( h_p \) attenuates quickly, the error function can be approximated by

\[ E_n \approx h_{L+M+1} T_{L+M+1}(\tilde{k}) \]  

(15)

\[ h_{L+M+1} = c_{L+M+1} + \frac{1}{2} \sum_{j=1}^{M} b_j c_{L+M+1} + c_{L+M-1} \]  

(16)

Hence \( R_{LM}(\tilde{k}) \) can be used as the best uniform rational approximation for \( I(k) \).

3 Numerical results

3.1 Numerical examples for PEC objects

As the first example, a PEC sphere with a radius of 0.3 cm is considered, and the surface of the sphere is discretised into 720 triangular elements. As shown in Fig. 1, the results obtained by the best uniform polynomial approximation performed by Chebyshev series with order of 7 agrees well with that of the direct solution for electric field integral.
equation (EFIE). When the AWE scheme is applied with its expansion point at 30 GHz, the Taylor series with the same order produces an accurate result only in a narrow frequency band ranging from 22 to 38 GHz, and the pade’ approximation obtains the accurate solution from 15 to 48 GHz. The CPU time required for direct solution, AWE and Chebyshev are 980, 89 and 92 s, respectively. It is shown that, even before transformed to be rational approximation, the Chebyshev series produces more accurate results than the AWE- pade’ scheme.

A target that is made up by two cubes illuminated by a pane wave propagating in the $z$-direction and $E$-polarised in the $x$-direction is considered. The two cubes are uniformly placed on $y$-axis, the length of the cube is chosen to be 1 cm, and the centre distance of them is 5 cm. As shown in Fig. 2, the accuracy of AWE (with expanded point at 18.5 GHz) will be severely limited by the radius of convergence in Taylor series, and the MBPE method which is applied with two sampled frequency points at 13 and 24 GHz gives a better result, whereas the Maehly method produces more accurate results over the entire frequency band. The surface of the two cubes is discretised into 1536 triangular elements. With a frequency step of 1 GHz, it takes the direct solution of EFIE 2494 s to obtain the solution from 2 to 35 GHz. With 0.1 GHz increments over the entire band, the AWE scheme, MBPE method and Maehly method consumed 589, 592 and 596 s, respectively.

To consider the application of the proposed method to the solution of combined field integral equation [14], a PEC sphere with a radius of 0.5 cm is considered. As we can see from Fig. 3, the Maehly method produces accurate result from 5 to 45 GHz, whereas the AWE scheme and MBPE method obtain a more narrow solution. The surface of the object is discretised into 980 triangular elements.

### 3.2 Numerical examples for homogenous dielectric objects

When considering the scattering problem of an arbitrarily shaped homogeneous dielectric body characterised by permittivity $\varepsilon_2$ and permeability $\mu_2$ and immersed in an infinite and homogeneous medium having permittivity $\varepsilon_1$ and permeability $\mu_1$, the so-called PMCHW equation [15], named after Poggio, Miller, Chang, Harrington, and Wu, who originally developed the formulation, is usually applied.

For a dispersive dielectric object, $\varepsilon_2$ and $\mu_2$ can be a function of frequency. For simplicity, the situation that only $\varepsilon_2$ varies with frequency is considered. The complex permittivity of a homogeneous dielectric can be described by the Debye model.

$$
\varepsilon_2(\omega) = \varepsilon'_2(\omega) + \frac{\varepsilon''_2(\omega) - \varepsilon'_2(\omega)}{1 + j \omega \tau_c}
$$

where $\varepsilon'_2(\omega)$ denotes the static dielectric constant, $\varepsilon''_2(\omega)$ the optical dielectric constant, and $\tau_c$ a relaxation time constant related to the original relaxation time constant $\tau$ by

$$
\tau_c = \frac{\tau \varepsilon'_2 + 2 \varepsilon_0}{\varepsilon''_2 + 2 \varepsilon_0}
$$

in which $\varepsilon_0$ denotes the permittivity of free space.

To compare the accuracy of the Chebyshev approximation with the Maehly approximation, a homogenous
Fig. 5  RCS frequency response of a dispersive dielectric sphere

Fig. 6  RCS frequency response of a dielectric cube with a side length of 1 cm

Fig. 7  Relative RMS RCS error of PEC sphere varies with the value of n
relative permeability is set to be $\mu_r = 1.0$. As we can see from Fig. 9, the experienced formula can also be applied to the analysis of homogeneous dielectric objects.

When multiple objects are considered and if the centre distance is so far that the interaction between them can be ignored, they should be treated as isolated objects when the experienced formula is applied, and the maximum of $n$ should be adopted.

5 Conclusion

An implementation of the best uniform rational approximation for MOM solution over a broad frequency band is proposed. The RCS of different PEC or dielectric objects are computed and compared with the direct solution method, MBPE method and AWE technique. Compared with the direct solution by MOM, the presented method is found to be superior in terms of the CPU time to obtain RCS frequency response. Compared with the MBPE method and AWE scheme, the presented technique can obtain accurate results over much broader frequency band without increasing any memory, and it is shown that the use of Maehly method can speed up the calculations as much as AWE or MBPE.

The accuracy of the proposed approximation approach and its relation to the order of the best polynomial series is preliminary discussed, and further research on it, especially for multiple objects, can be the topic of interest for future work.

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7 References