Modeling of Nonlinear Response from Metallic Metamaterials by Maxwell-Hydrodynamic Equations

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OUTLINE

- **Background for Nonlinear Optics**
- **Maxwell-Hydrodynamic Framework**
- **Theoretical Results**
  - **Charge Conservation**
  - **Energy Conservation**
  - **Angular Momentum Conservation**
  - **Parity Conservation**
- **Conclusion**

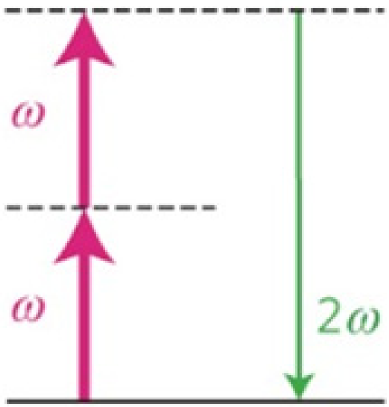
NONLINEAR OPTICS: A QUICK REVIEW

$$\mathbf{P} = \varepsilon_0 \left[\chi^{(1)} \cdot \mathbf{E} + \underbrace{\chi^{(2)} : \mathbf{EE} + \chi^{(3)} : \mathbf{EEE} + \cdots}_{\text{nonlinear response}} \right] \qquad (1)$$

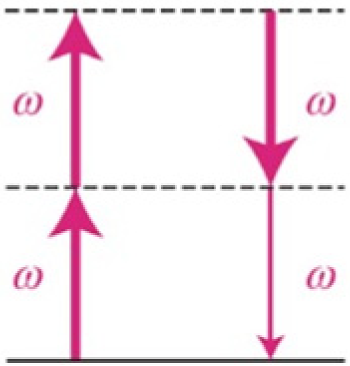
nonlinear response

SHG, THG, FWM, Kerr effect …

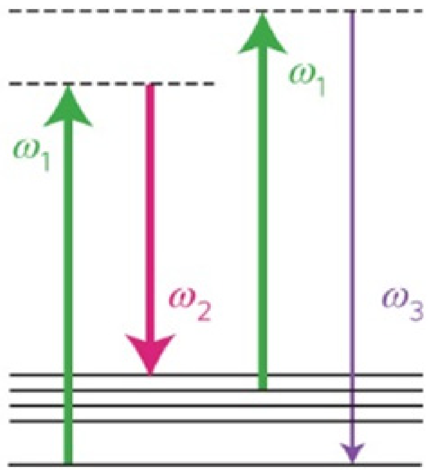
Linear and nonlinear susceptibility: $\chi^{(1)} \gg \chi^{(2)} \gg \chi^{(3)} \gg \cdots$



second harmonic generation



four wave mixing

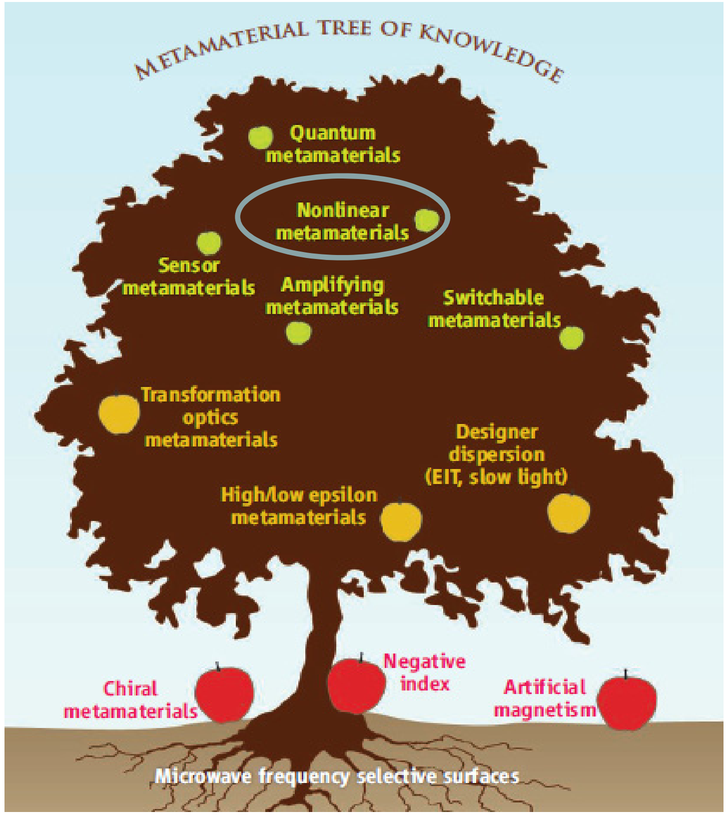


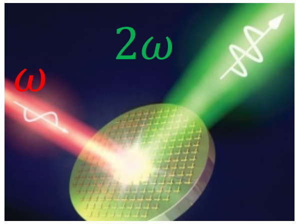
anti-Stokes Raman scattering

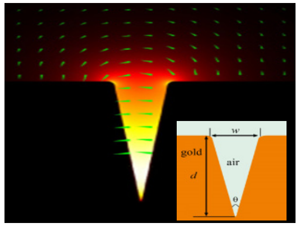
Martti Kauranen and Anatoly V. Zayats, *Nature Photonics* **6**: 737–748, 2012.

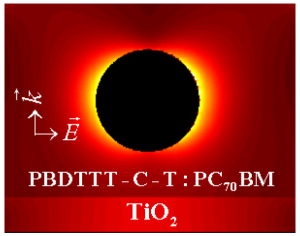
WHY NONLINEAR PLASMONICS?

- Support localized surface plasmon resonance
- Strong near field enhancement
- Confine energy in a small volume
- Amplify nonlinear processes









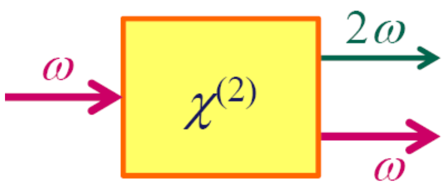
Control of amplitude, phase, polarization, and wavefront of electromagnetic waves by nonlinear metamaterials and plasmonics is an emerging research direction, which is full of challenges in mathematical modeling and physical designs.

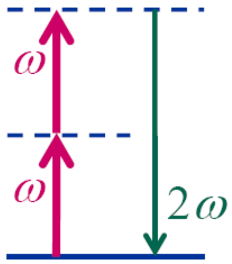
N. I. Zheludev, The Road Ahead for Metamaterials, *Science*, **328**: 582–583, 2010.

WHY SHG IS SPECIAL?

Second-harmonic generation (SHG)

$$P_i = \sum_{j,k} \chi_{ijk}^{(2)} E_j E_k$$





Parametric nonlinear process
- charge conservation
- energy conservation
- angular momentum conservation
- parity conservation

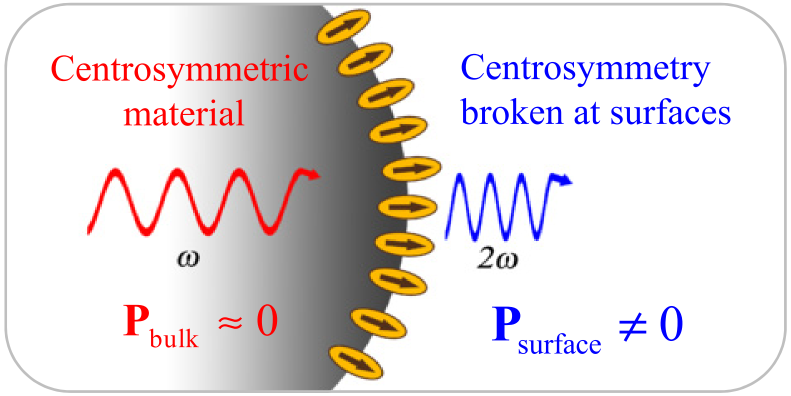
Selection Rules

Surface SHG in plasmonic nanostructures

- sensitive to particle shape
- sensitive to environment
- sensitive to the incident beam property
- strong nonlinear polarization

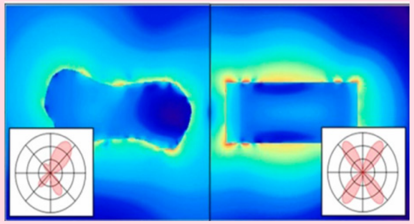
$\mathbf{r} \to -\mathbf{r}$ $\quad \mathbf{E} \to -\mathbf{E}$ $\quad \mathbf{P} \to -\mathbf{P}$

$-\mathbf{P} = \chi^{(2)} : (-\mathbf{E})^2 = \chi^{(2)} : \mathbf{E}^2 = \mathbf{P}$ $\Rightarrow$ $\boxed{\mathbf{P} = 0}$

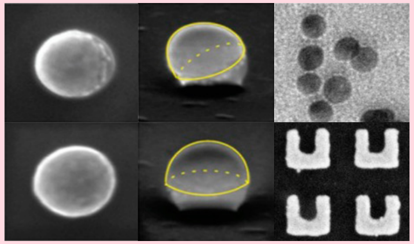


APPLICATIONS

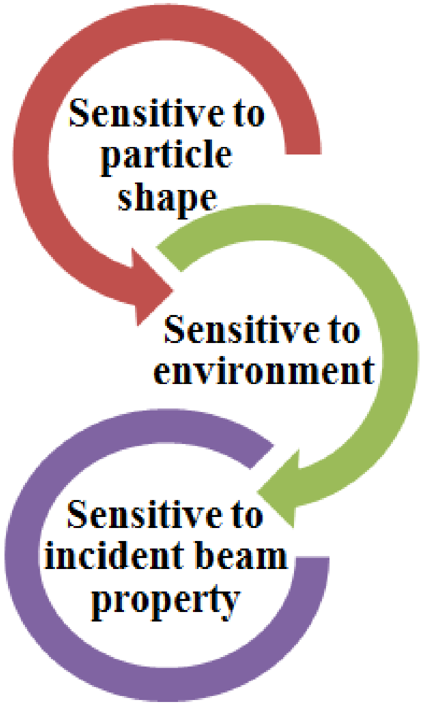
ultrasensitive shape characterization



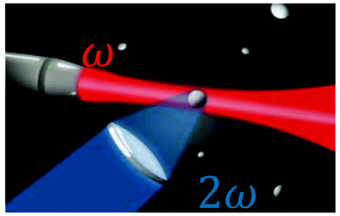
Martin, Nano Lett. (2013)



Zhang, Nano Lett. (2011)

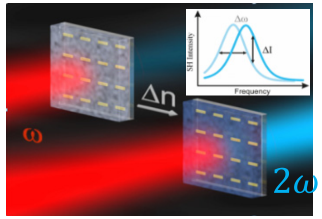


single nanoparticle detection

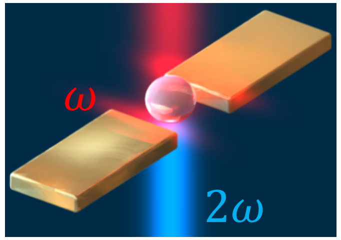


Butet, Nano Lett.(2012)

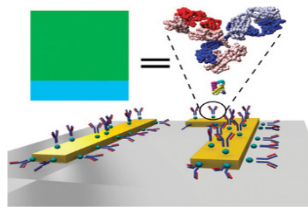
nonlinear plasmonic sensing



Mesch, Nano Lett. (2016)

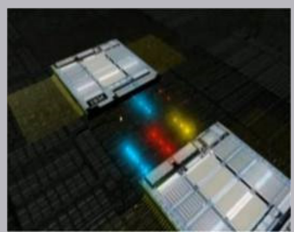


Butet, ACS Nano (2014)



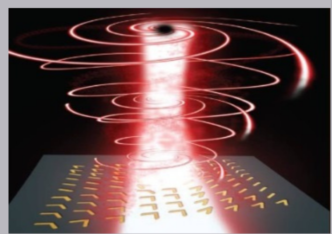
Yu, Science (2012)

all optical signal processing



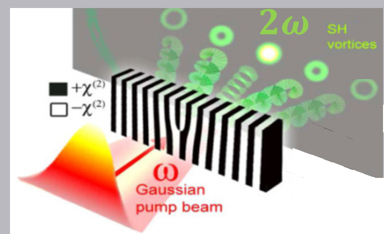
All optical chip, IBM (2011)

OAM wavefront generation



Capasso, Science (2011)

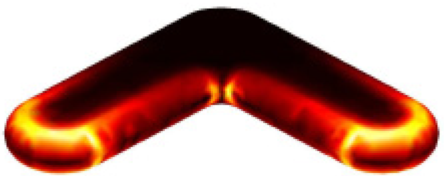
new frequency generation

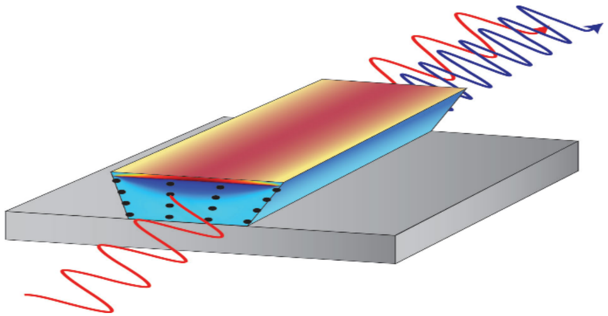


Bloch, Phys. Rev. Lett.(2012)

HYDRODYNAMIC MODEL

When electromagnetic waves strongly interact with metallic structures, it can couple to free electrons near the metal surface resulting in complex linear and nonlinear responses. Interestingly, the complex motion of electrons within metallic structures resembles that of fluids governed by the same hydrodynamic equation.





Maxwell

$$\varepsilon_0 \frac{\partial \boldsymbol{E}}{\partial t} = \nabla \times \boldsymbol{H} - \frac{\partial \boldsymbol{P}}{\partial t}$$

$$\mu_0 \frac{\partial \boldsymbol{H}}{\partial t} = -\nabla \times \boldsymbol{E}$$

Hydrodynamic

$$m_e n \left(\frac{\partial \boldsymbol{v}}{\partial t} + \boxed{\boldsymbol{v} \cdot \nabla \boldsymbol{v}} \right) + \gamma m_e n \boldsymbol{v} = -q_e n (\boldsymbol{E} + \boxed{\boldsymbol{v} \times \mu_0 \boldsymbol{H}})$$

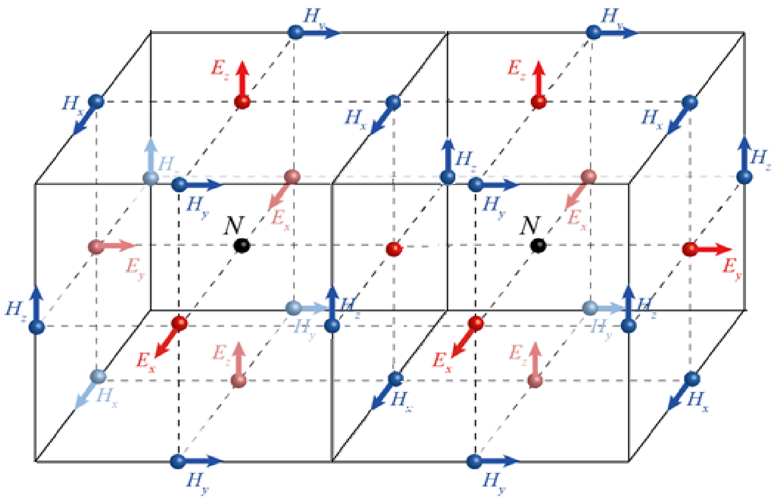
Continuity

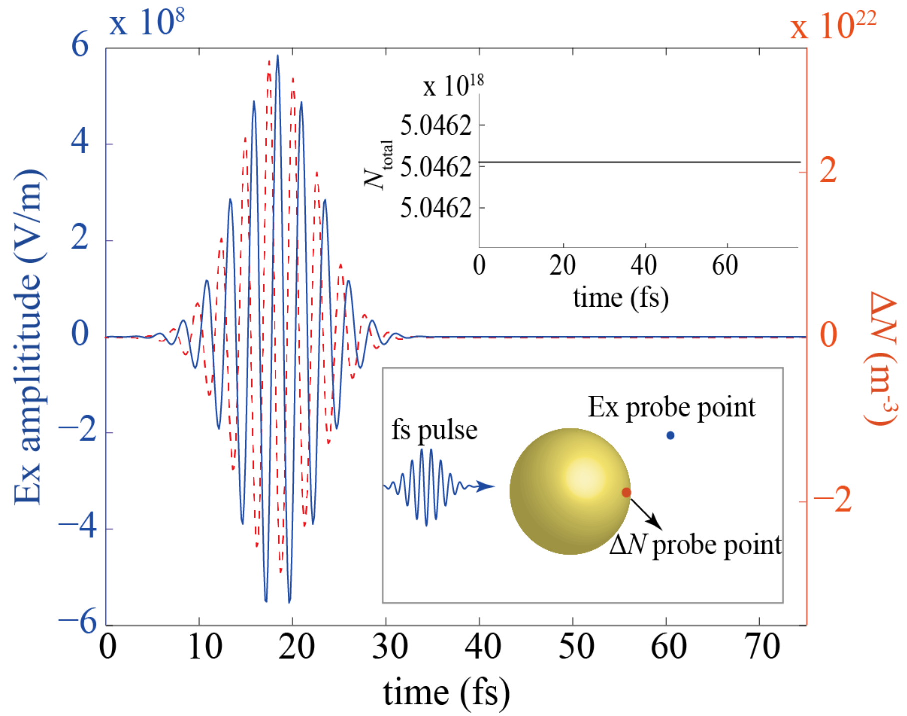
$$\frac{\partial n}{\partial t} + \nabla \cdot \boxed{(n\boldsymbol{v})} = 0$$

$$\frac{\partial \boldsymbol{P}}{\partial t} = -q_e n \boldsymbol{v}$$

The equation is solved by FDTD method with Yee grids.

CHARGE CONSERVATION

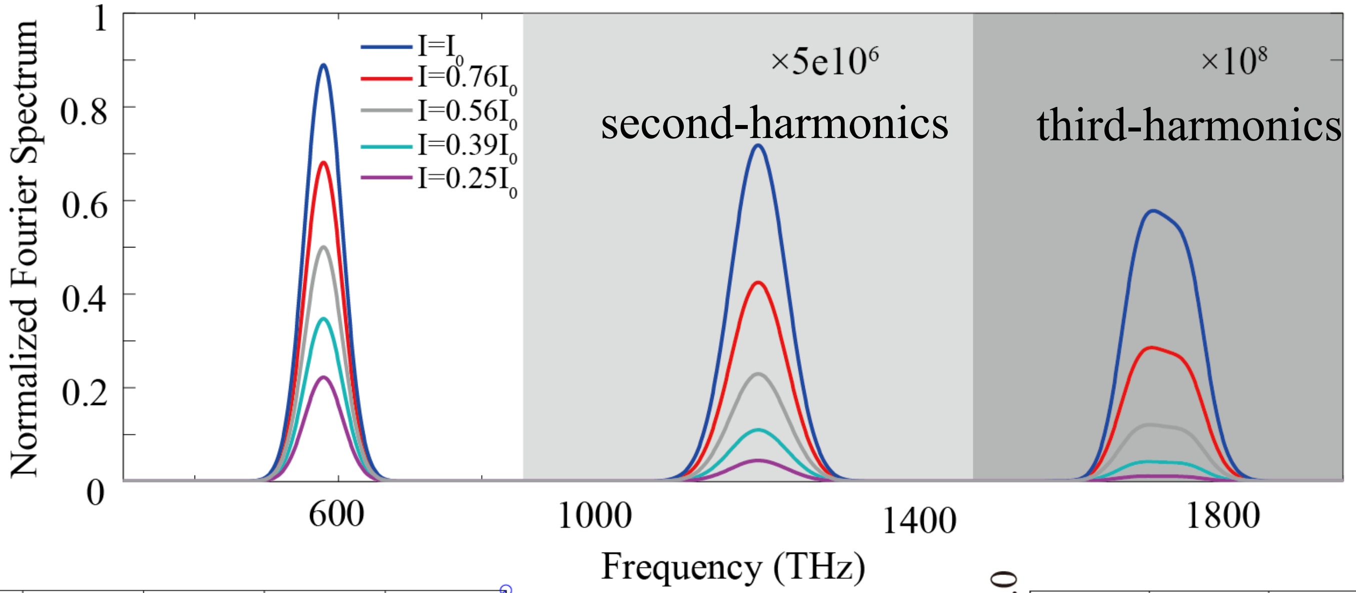


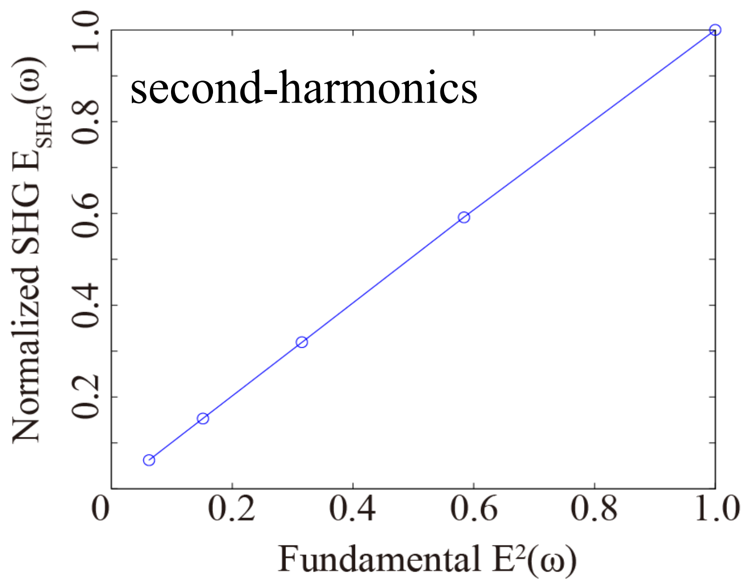


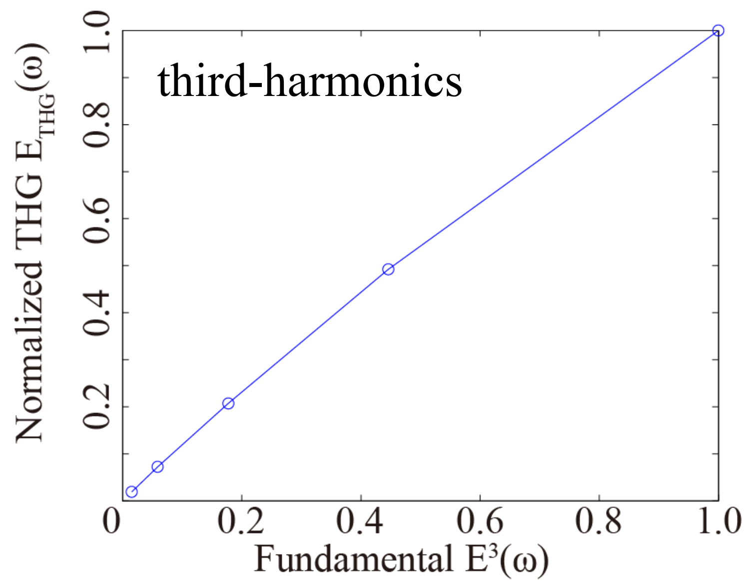
At each point, the electron density fluctuates. However, the total charge within the sphere is conserved.

$$
\begin{aligned}
& n^{l+3/2}(i+1/2, j+1/2, k+1/2) = n^{l+1/2}(i+1/2, j+1/2, k+1/2) \\
& -\Bigg[\frac{\Delta t}{\Delta x}\Big(\overline{n}^{l+1/2}(i+1, j+1/2, k+1/2)v_x^{l+1}(i+1, j+1/2, k+1/2) - \overline{n}^{l+1/2}(i, j+1/2, k+1/2)v_x^{l+1}(i, j+1/2, k+1/2)\Big) \\
& +\frac{\Delta t}{\Delta y}\Big(\overline{n}^{l+1/2}(i+1/2, j+1, k+1/2)v_y^{l+1}(i+1/2, j+1, k+1/2) - \overline{n}^{l+1/2}(i+1/2, j, k+1/2)v_y^{l+1}(i+1/2, j, k+1/2)\Big) \\
& +\frac{\Delta t}{\Delta z}\Big(\overline{n}^{l+1/2}(i+1/2, j+1/2, k+1)v_z^{l+1}(i+1/2, j+1/2, k+1) - \overline{n}^{l+1/2}(i+1/2, j+1/2, k)v_z^{l+1}(i+1/2, j+1/2, k)\Big)\Bigg]
\end{aligned}
$$

ENERGY CONSERVATION/CONVERSION







ANGULAR MOMENTUM CONSERVATION (1)

- For a metallic structure with <u>*N*-fold rotational symmetry</u>

The relation between the spin and orbital angular momenta of the incident field and the total angular momenta of the second harmonic field is given by

$$-\upsilon + 2(l + s) = nN$$

s: spin angular momentum of the incident field (1 or -1)

l: orbital angular momentum of the incident field

υ: total angular momentum of the second-harmonic field

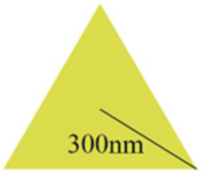
$n = 0, 1, 2, \ldots$ is an integer

ANGULAR MOMENTUM CONSERVATION (2)

Unit cell

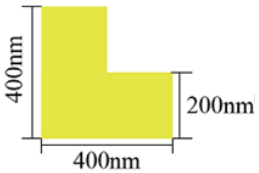
$$-v + 2s = nN$$

l=0



triangle-shape

- N=3, (v, s) = (+1,-1) and (-1,+1)
- Polarization state of the second-harmonic wave is always opposite to that of the fundamental wave.



L-shape

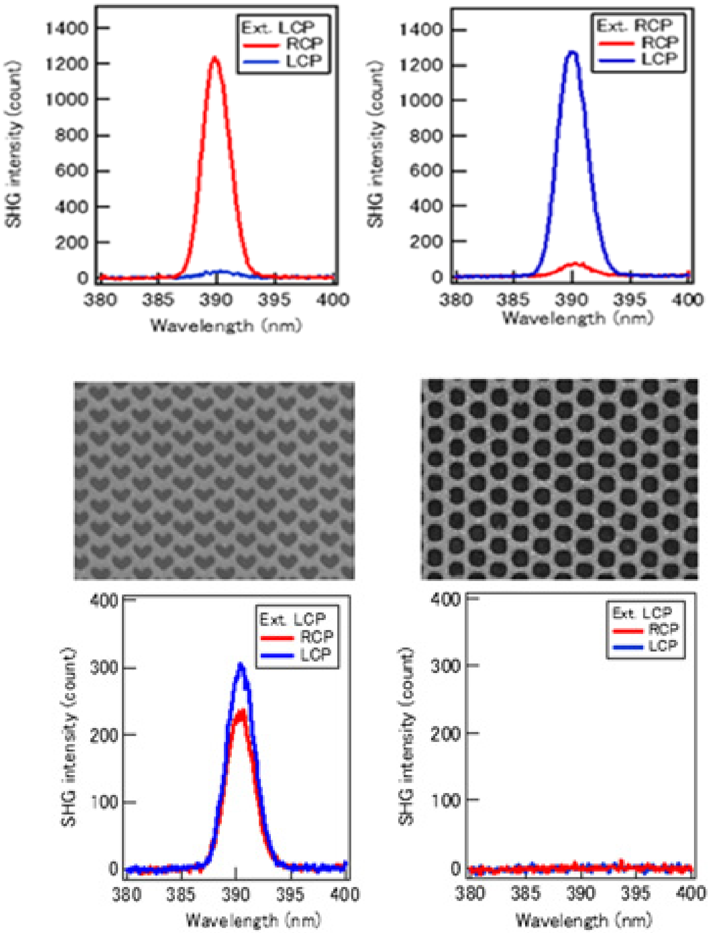
- N=1, the identity is satisfied for any v , s.

disk-shape

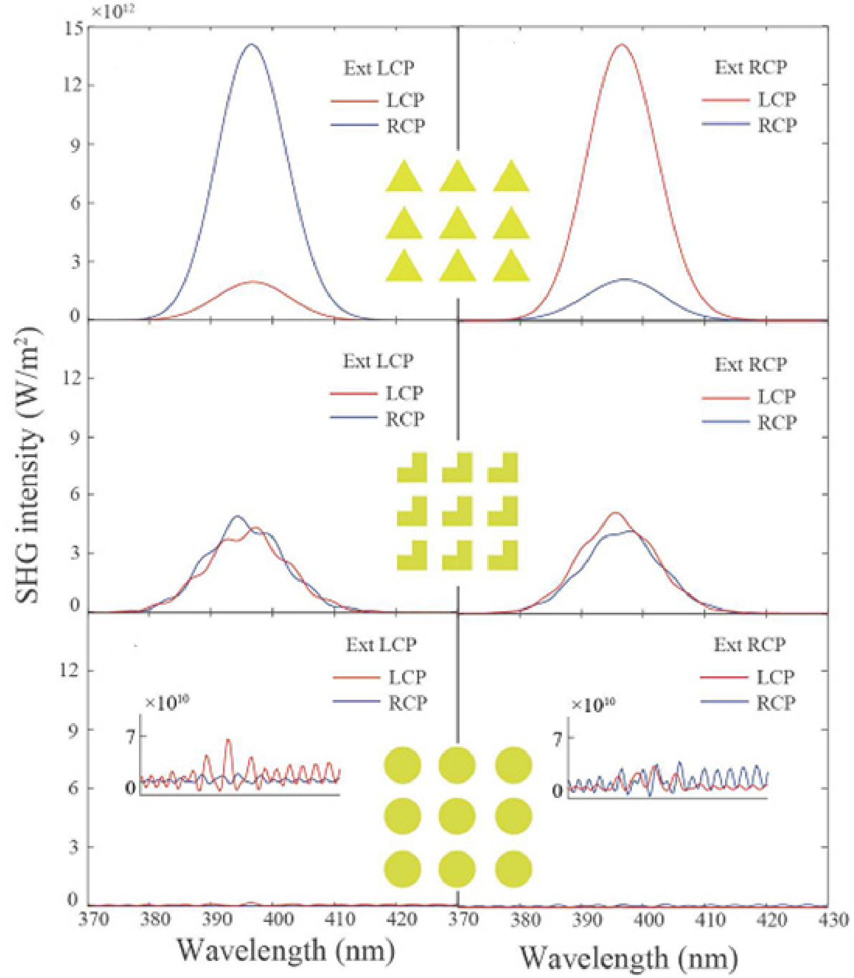
- N=2 or N>3, the identity is not satisfied by any sets of v, s
- Second harmonic generation is forbidden or very weak.

ANGULAR MOMENTUM CONSERVATION (3)

experimental



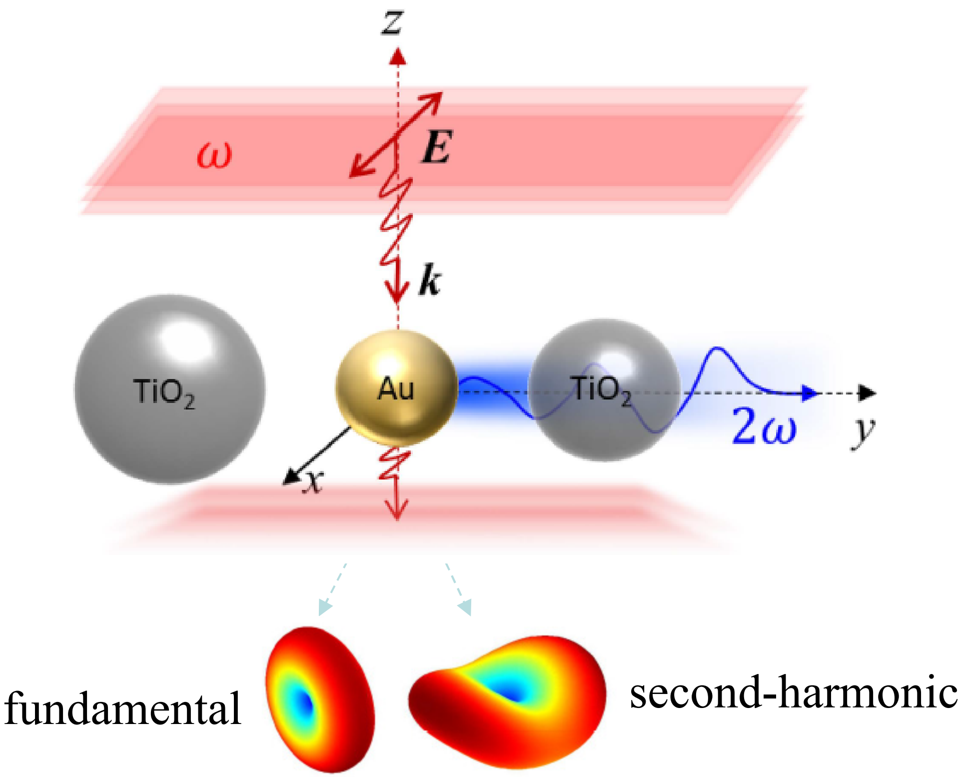
numerical

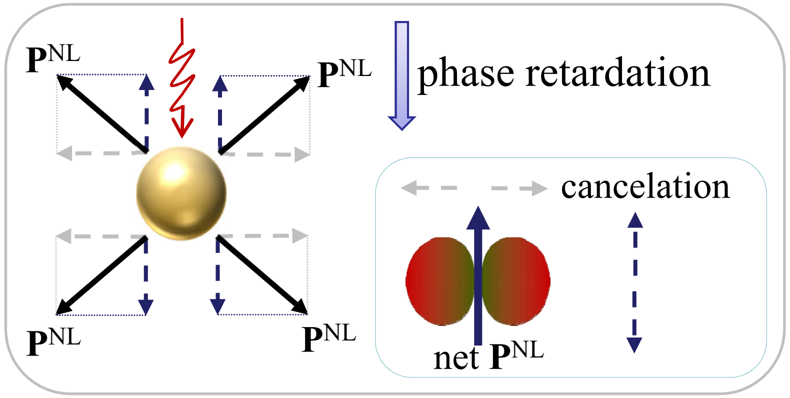


K. Konishi, T. Higuchi, J. Li, J. Larsson, S. Ishii, and M. K.-Gonokami, *Phy. Rev. Lett.*, **112**: 135502, 2014.

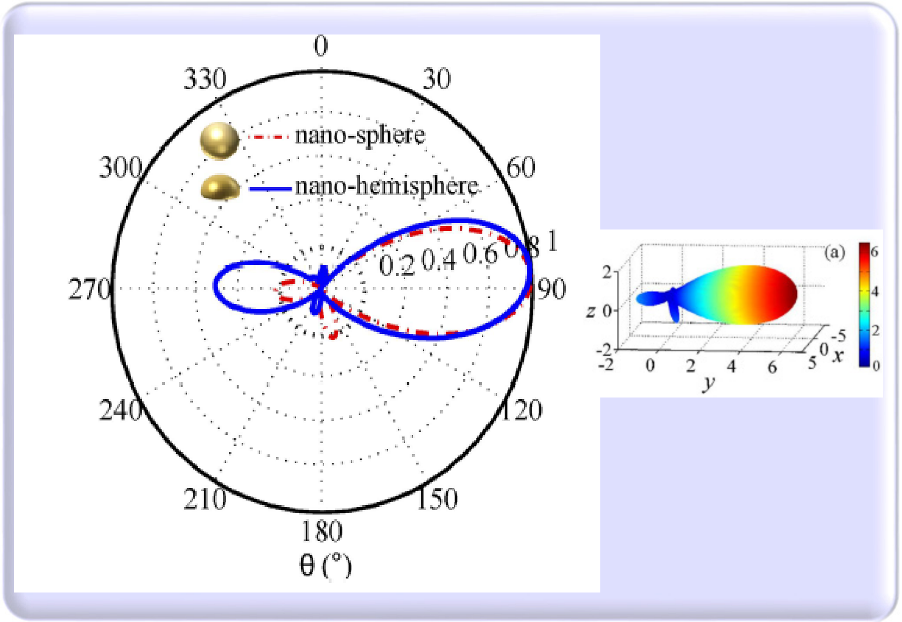
PARITY CONSERVATION (1)

Compact nonlinear Yagi-Uda nanoantenna



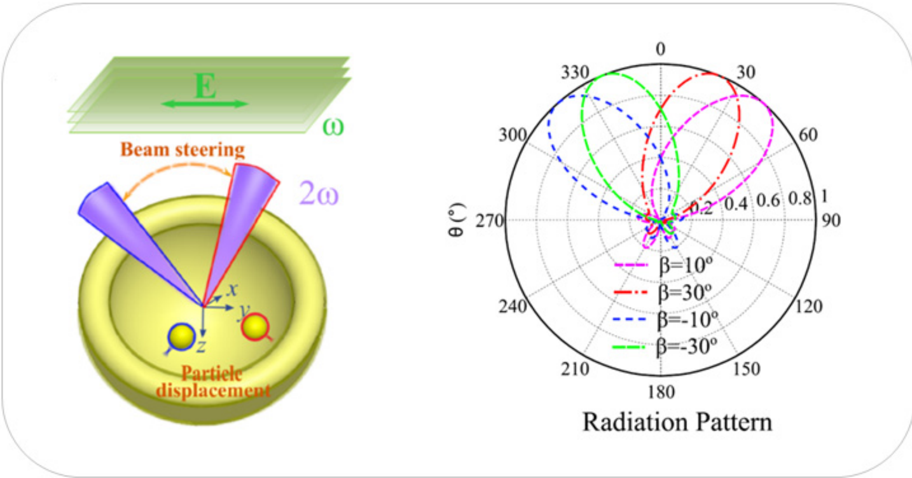


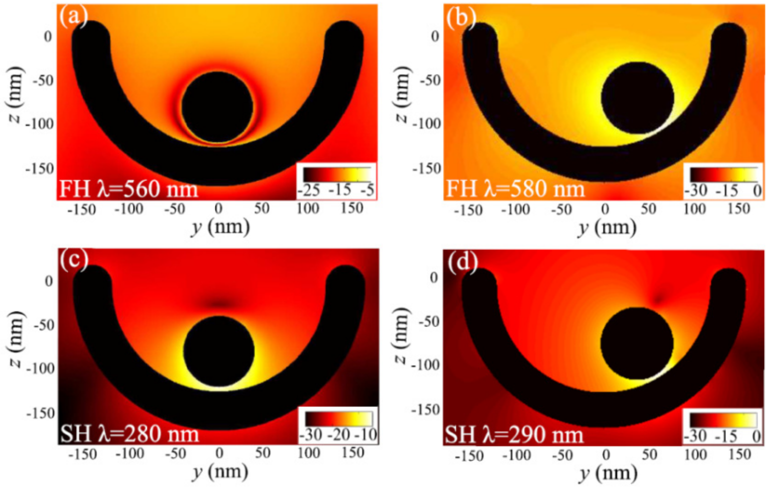
Second-harmonic radiation from an object is strictly zero along the incident z direction, if the projection of the object onto the xoy plane is centrosymmetric.

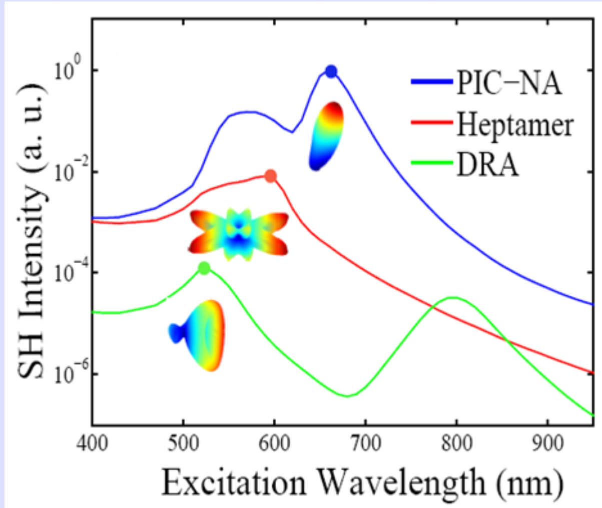


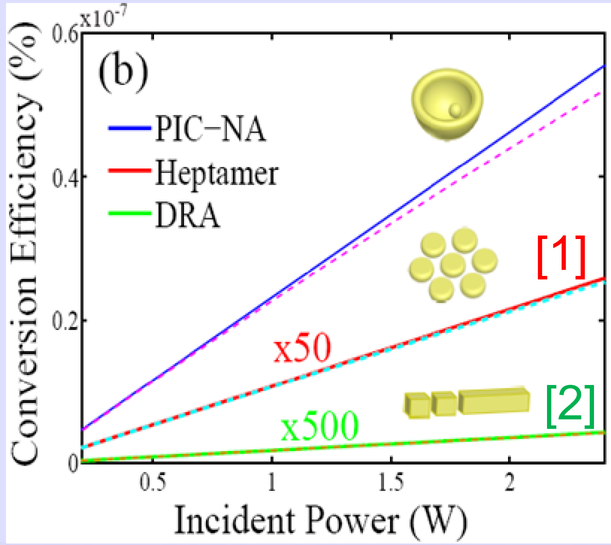
PARITY CONSERVATION (2)

Particle-in-cavity nanoantenna









[1] K. Thyagarajan, J. Butet, O. J. F. Martin, *Nano Lett.* **13**: 1847–1851, 2013
[2] K. Thyagarajan, S. Rivier, A. Lovera, and O. J. F. Martin, *Opt. Express* **20**, 12860–12865, 2012.

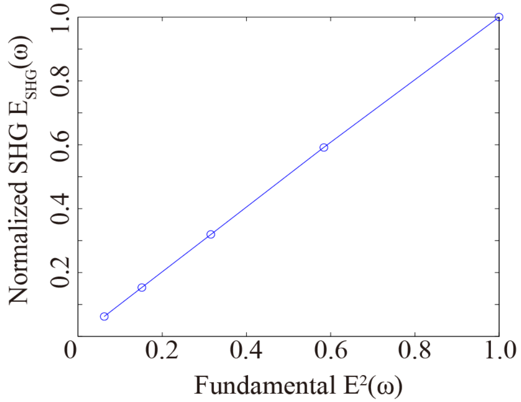
PUBLICATIONS

1. Xiaoyan Y.Z. Xiong, Li Jun Jiang, Wei E.I. Sha, Yat Hei Lo, and Weng Cho Chew, "Compact Nonlinear Yagi-Uda Nanoantennas," *Scientific Reports*, vol. 6, pp. 18872, Jan. 2016.

2. Ming Fang, Zhixiang Huang, Wei E. I. Sha, Xiaoyan Y. Z. Xiong, and Xianliang Wu, "Full Hydrodynamic Model of Nonlinear Electromagnetic Response in Metallic Metamaterials（Invited Paper）," Progress In Electromagnetics Research, vol. 157, 63-78, Oct. 2016.

3. Xiaoyan Y.Z. Xiong, Li Jun Jiang, Wei E.I. Sha, Yat Hei Lo, Ming Fang, Weng Cho Chew, and Wallace C.H. Choy, "Strongly Enhanced and Directionally Tunable Second-Harmonic Radiation by a Plasmonic Particle-in-Cavity Nanoantenna," Physical Review A, vol. 94, no. 5, pp. 053825, Nov. 2016.

CONCLUSION



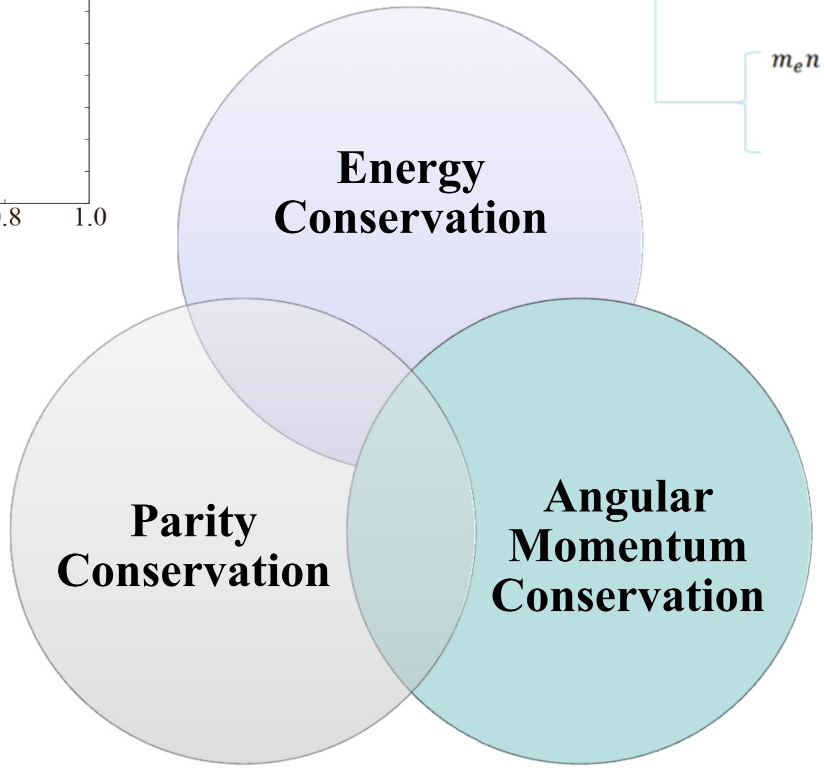
$$\varepsilon_0 \frac{\partial \boldsymbol{E}}{\partial t} = \nabla \times \boldsymbol{H} - \frac{\partial \boldsymbol{P}}{\partial t}$$

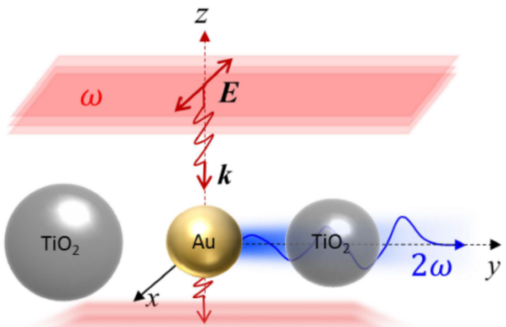
$$\mu_0 \frac{\partial \boldsymbol{H}}{\partial t} = -\nabla \times \boldsymbol{E}$$

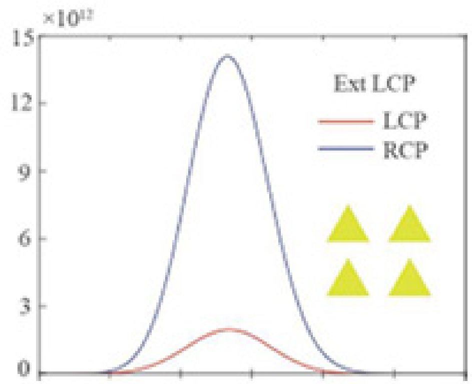
$$m_e n \left(\frac{\partial \boldsymbol{v}}{\partial t} + \boxed{\boldsymbol{v} \cdot \nabla \boldsymbol{v}} \right) + \gamma m_e n \boldsymbol{v} = -q_e n (\boldsymbol{E} + \boxed{\boldsymbol{v} \times \mu_0 \boldsymbol{H}})$$

$$\frac{\partial n}{\partial t} + \nabla \cdot \boxed{(n\boldsymbol{v})} = \boldsymbol{0}$$

$$\frac{\partial \boldsymbol{P}}{\partial t} = -q_e n \boldsymbol{v}$$







ACKNOWLEDGEMENT

Any Questions and Discussions?