Modeling of Nonlinear Response from Metallic Metamaterials by Maxwell-Hydrodynamic Equations

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OUTLINE

- Background for Nonlinear Optics
- Maxwell-Hydrodynamic Framework
- Theoretical Results
  - Charge Conservation
  - Energy Conservation
  - Angular Momentum Conservation
  - Parity Conservation
- Conclusion
NONLINEAR OPTICS: A QUICK REVIEW

\[ P = \varepsilon_0 \left[ \chi^{(1)} \cdot E + \chi^{(2)} : EE + \chi^{(3)} : EEE + \cdots \right] \]  \hspace{1cm} (1)

nonlinear response
SHG, THG, FWM, Kerr effect …

Linear and nonlinear susceptibility: \( \chi^{(1)} \gg \chi^{(2)} \gg \chi^{(3)} \gg \cdots \)

second harmonic generation four wave mixing anti-Stokes Raman scattering

WHY NONLINEAR PLASMONICS?

- Support localized surface plasmon resonance
- Strong near field enhancement
- Confine energy in a small volume
- Amplify nonlinear processes

Control of amplitude, phase, polarization, and wavefront of electromagnetic waves by nonlinear metamaterials and plasmonics is an emerging research direction, which is full of challenges in mathematical modeling and physical designs.

**WHY SHG IS SPECIAL?**

**Second-harmonic generation (SHG)**

\[ P_i = \sum_{j,k} \chi^{(2)}_{ijk} E_j E_k \]

**Surface SHG in plasmonic nanostructures**

- sensitive to particle shape
- sensitive to environment
- sensitive to the incident beam property
- strong nonlinear polarization

Parametric nonlinear process
- charge conservation
- energy conservation
- angular momentum conservation
- parity conservation

Selection Rules

Centrosymmetric material

\[ P_{\text{bulk}} \approx 0 \]

Centrosymmetry broken at surfaces

\[ P_{\text{surface}} \neq 0 \]
APPLICATIONS

ultrasensitive shape characterization

Sensitive to particle shape

Sensitive to environment

Sensitive to incident beam property

single nanoparticle detection

nonlinear plasmonic sensing


Butet, ACS Nano (2014)

Yu, Science (2012)

zhang, Nano Lett. (2011)

Martin, Nano Lett. (2013)

all optical signal processing

OAM wavefront generation

new frequency generation

All optical chip, IBM (2011)

Capasso, Science (2011)

HYDRODYNAMIC MODEL

When electromagnetic waves strongly interact with metallic structures, it can couple to free electrons near the metal surface resulting in complex linear and nonlinear responses. Interestingly, the complex motion of electrons within metallic structures resembles that of fluids governed by the same hydrodynamic equation.

The equation is solved by FDTD method with Yee grids.
At each point, the electron density fluctuates. However, the total charge within the sphere is conserved.
ENERGY CONSERVATION/CONVERSION

![Graph showing normalized Fourier spectrum for second harmonics and third harmonics.](image)

- Second harmonics
  - Frequency range: 600 THz to 1800 THz
  - Normalized Fourier Spectrum graph

- Third harmonics
  - Frequency range: 700 THz to 1900 THz
  - Normalized Fourier Spectrum graph

- Normalized SHG $E_{\text{SHG}}(\omega)$ vs. Fundamental $E^2(\omega)$
  - Second harmonics
  - Third harmonics

- Normalized THG $E_{\text{THG}}(\omega)$ vs. Fundamental $E^3(\omega)$
  - Second harmonics
  - Third harmonics
ANGULAR MOMENTUM CONSERVATION (1)

- For a metallic structure with $N$-fold rotational symmetry

The relation between the spin and orbital angular momenta of the incident field and the total angular momenta of the second harmonic field is given by

$$-v + 2(l + s) = nN$$

$s$: spin angular momentum of the incident field (1 or $-1$)

$l$: orbital angular momentum of the incident field

$v$: total angular momentum of the second-harmonic field

$n = 0, 1, 2, ...$ is an integer
ANGULAR MOMENTUM CONSERVATION (2)

\[ -\nu + 2s = nN \]

- \( N=2 \) or \( N>3 \), the identity is not satisfied by any sets of \( \nu \), \( s \).
- \( N=3 \), \( (\nu, s) = (+1, -1) \) and \( (-1, +1) \)
- Polarization state of the second-harmonic wave is always opposite to that of the fundamental wave.
- \( N=1 \), the identity is satisfied for any \( \nu \), \( s \).
- \( N=2 \) or \( N>3 \), the identity is not satisfied by any sets of \( \nu \), \( s \)
- Second harmonic generation is forbidden or very weak.
ANGULAR MOMENTUM CONSERVATION (3)

PARITY CONSERVATION (1)

Compact nonlinear Yagi-Uda nanoantenna

Second-harmonic radiation from an object is strictly zero along the incident $z$ direction, if the projection of the object onto the $xoy$ plane is centrosymmetric.
PARITY CONSERVATION (2)

Particle-in-cavity nanoantenna


PUBLICATIONS


CONCLUSION

Energy Conservation

Parity Conservation

Angular Momentum Conservation

\[ \varepsilon_0 \frac{\partial E}{\partial t} = \nabla \times H - \frac{\partial P}{\partial t} \]
\[ \mu_0 \frac{\partial H}{\partial t} = -\nabla \times E \]

\[ m_e n \left( \frac{\partial v}{\partial t} + v \cdot \nabla v \right) + \gamma m_e n v = -q_e n (E + \nu \times \mu_0 H) \]

\[ \frac{\partial n}{\partial t} + \nabla \cdot (n \nu) = 0 \]
\[ \frac{\partial P}{\partial t} = -q_e n v \]
ACKNOWLEDGEMENT

Any Questions and Discussions?