mm, $\lambda_1 + \lambda_2 = mm$, $\lambda_h = 2.4 mm$, $W_{c1} = 0.2 mm$, $\lambda_c = 9.5 mm$, $W_c = 5 mm$, and $g = 0.4 mm$. Figure 4 depicts the simulated results of the conventional three-stage PC-BPF and the proposed filters with $\lambda_2 = 0$ to 3 mm. It exhibits that for the proposed filter the growing second harmonic was significantly suppressed and the rejection bandwidth was extended by increasing the value $\lambda_2$. As the measured results in Figure 5, for interlaced length $\lambda_2 = 0$ mm the second harmonic suppression was from $-19$ to $-53 dB$ when compared with the conventional PC-BPF. Moreover, its insertion loss and return loss at center frequency are $-1.5$ and $-19$ dB, respectively. In addition, the $-20$ dB rejection bandwidth was even achieved $4.5$ GHz with the interlaced length $\lambda_2 = 3$ mm that stands for $40\%$ improvement.

4. CONCLUSION

A wide rejection bandwidth bandpass filter with SIH resonator and interlaced coupled-line has been proposed in this paper. This filter was described significantly; it improves the performance of conventional parallel coupled-line structure. The interlaced coupled-line can extend the rejection bandwidth for $40\%$ improvement, and the SIH resonator can not only suppress the $2f_0$ spurious but also steepen the upper roll off characteristic. Furthermore, its insertion loss and return loss can be maintained at an acceptable level. With implementation and measurement, it is verified for our design concept.

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OPTIMAL SYMPLECTIC INTEGRATORS FOR NUMERICAL SOLUTION OF TIME-DOMAIN MAXWELL’S EQUATIONS

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ABSTRACT: Optimal symplectic integrators were proposed to improve the accuracy in numerical solution of time-domain Maxwell’s equations. The proposed symplectic scheme has almost the same stability and numerical dispersion as the mostly used fourth-order symplectic scheme, but acquires more efficiency in the calculations at the same computational cost. © 2007 Wiley Periodicals, Inc. Microwave Opt Technol Lett 49: 545–547, 2007; Published online in Wiley InterScience (www.interscience.wiley.com). DOI 10.1002/mop.22193

Key words: symplectic integrators; stability; numerical dispersion

1. INTRODUCTION

Symplectic integrator schemes show substantial benefits in the fields of physics ranging from classical mechanics, electromagnetism, and quantum mechanics [1–3]. In this article, we obtained the optimal symplectic integrators based on the similar approach as presented in Ref. 4.

First, consider the following general system of ordinary differential equations

$$\frac{du}{dt} = (A + B)u, \quad 0 \leq t < +\infty \quad (1)$$

We can write the mapping from $t = 0$ to $t = \Delta t$, as evolution equations of the form

$$u(t) = \exp(\Delta t(A + B))u(0) \quad (2)$$

where $A$ and $B$ are noncommuting operators, which can be solved by approximating $\exp(\Delta t(A + B))$ to the $n$th-order in the product form

$$\exp(\Delta t(A + B)) = \prod_{p=1}^{n} \exp(\Delta tD_pB) \exp(\Delta tC_pA) + O((\Delta t)^{n+1}) \quad (3)$$

via a well chosen set of factorization or decomposition coefficients $\{C_p\}$ and $\{D_p\}$, $m$ and $n$ ($m \geq n$) are the number of stages and the order of scheme, respectively. Generally, Eq. (1) results in a class of factorized symplectic integrators scheme. To determine the coefficients $C_p$ and $D_p$, we expand the left hand side and right hand side of Eq. (3) in powers of $\Delta t$. The coefficients $C_p$ and $D_p$ of the $n$th-order scheme are determined, when we want the two expressions to agree up to $(\Delta t)^m$. In this article, we will consider only the symmetric factorization scheme such that $C_p = C_{m-p+1} = -p(0 < p < m + 1)$, $D_p = D_{m-p}(0 < p < m)$ and $D_m = 0$. The scheme is exactly time reversible [1]. Here, we consider an extended factorization scheme of the fourth-order by allowing one more stage than the minimum stage, i.e., five-stage integrator.

We perform the similar approach as indicated in Ref. 4 based on the minimization of the truncation error-function to find the coefficients. The coefficients are finally given in Table 1. The norm of

![Table 1 Coefficients of the Symplectic Integrators](https://example.com/Table1.png)

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the fifth-order error is 0.00061 in the optimal scheme when compared with 0.0069 of the scheme in Refs. 2 and 3. Thus, the optimal scheme is more accuracy.

2. STABILITY AND DISPERSION OF OPTIMAL SYMPLECTIC SCHEME

Maxwell’s equations in an isotropic, lossless, and source-free medium can be written in a matrix form as

$$\frac{\partial}{\partial t} \begin{bmatrix} H \\ E \end{bmatrix} = (A + B) \begin{bmatrix} H \\ E \end{bmatrix}$$

(4)

$$A = \begin{bmatrix} \{0\}_{3 \times 3} - \mu^{-1} R & \{0\}_{3 \times 3} \\ \{0\}_{3 \times 3} & \{0\}_{3 \times 3} \end{bmatrix}, \quad R = \begin{bmatrix} \{0\}_{3 \times 3} & \{0\}_{3 \times 3} \\ \{0\}_{3 \times 3} & \{0\}_{3 \times 3} \end{bmatrix}$$

(5)

where $\mu$ and $\varepsilon$ is the permeability and permittivity, $\{0\}_{3 \times 3}$ is the 3 x 3 null matrix, $R$ is the 3 x 3 matrix representing the curl operator. As indicated in Refs. 2 and 3, we use optimal symplectic integrators to discretize Maxwell’s equations in the time direction and then use a fourth-order accuracy to discretize $R$ in the space direction.

For the scheme to be stable, $|s| \leq 1$. The $s$ can be written as

$$s = 1 + \left(\frac{1}{2}\right) \sum_{p=1}^{5} g_p \left( \frac{1}{\mu_e} \Delta \tau (\eta_1^2 + \eta_2^2 + \eta_3^2) \right)$$

(6)

$$g_p = \sum_{i_1 < i_2 < \ldots < i_p} C_{i_1}D_{j_1}C_{i_2}D_{j_2} \cdots C_{i_p}D_{j_p}$$

(7)

$$\eta_i = \frac{27(e^{-j\delta\Delta x^2} - e^{j\delta\Delta x^2}) - (e^{-j\delta\Delta y^2} - e^{j\delta\Delta y^2})}{24\Delta x}$$

(8)

$$\eta_y = \frac{27(e^{-j\delta\Delta y^2} - e^{j\delta\Delta y^2}) - (e^{-j\delta\Delta x^2} - e^{j\delta\Delta x^2})}{24\Delta y}$$

(9)

where $k$ is the wave number, $\phi$ and $\theta$ are the wave propagating angles with respect to $x$- and $y$-axis, respectively. In our numerical simulations, we use space discretizations grid $\Delta x = \Delta y = \Delta$ and defined

$$\text{CFL} = \frac{\Delta}{\sqrt{\Delta x \Delta y}}$$

(14)

The stability of the scheme is determined by the maximum of CFL, i.e., CFL_{max}. The numerical results for CFL_{max} are also shown in Table 1. The optimal fourth-order scheme has almost the same CFL_{max} as the scheme in Ref. 3, but larger than the second-order method in Ref. 4. The numerical dispersion can be expressed as

$$\cos (\omega \Delta t) = s$$

(15)
scheme, standard FDTD, and FDTD (2, 4). Also, both fourth-order schemes permit a coarser discretization than other schemes for a given error bound, which, in turn, allow faster computation time and the storage of less data.

3. CONCLUSION

The proposed symplectic scheme is more efficient in the calculations at the same computational cost than the mostly used fourth-order symplectic scheme in Refs. 2 and 3, and provides a new approach for solving large computational domain electrodynamics problems.

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