Compact Nonlinear Yagi-Uda Nanoantennas

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Supporting Information

Supplementary Note 1: Multipole Expansion for Equivalent Currents

According to Huygens' principle, electric field can be represented by the Green's tensor $\overline{\mathbf{G}}$

$$\mathbf{E} = i\omega\mu \int \overline{\mathbf{G}}\left(\mathbf{r},\mathbf{r}'\right)\cdot\mathbf{J}\left(\mathbf{r}'\right)d\mathbf{r}' - \int \nabla\times\overline{\mathbf{G}}\left(\mathbf{r},\mathbf{r}'\right)\cdot\mathbf{M}\left(\mathbf{r}'\right)d\mathbf{r}' \tag{1}$$

where $\overline{\mathbf{G}}\left(\mathbf{r},\mathbf{r}'\right) = (\bar{\mathbf{I}} + \frac{\nabla\nabla}{k^2})g\left(\mathbf{r},\mathbf{r}'\right)$ and $g\left(\mathbf{r},\mathbf{r}'\right) = \frac{\exp(ik|\mathbf{r}-\mathbf{r}'|)}{4\pi|\mathbf{r}-\mathbf{r}'|}$.

In the far-field region ($k\mathbf{r} \gg 1$), it is sufficient to approximate

$$|\mathbf{r}-\mathbf{r}'| \approx r - \hat{\mathbf{r}}\cdot\mathbf{r}' \tag{2}$$

Then electric field is

$$\begin{aligned}\mathbf{E} &= i\omega\mu\frac{e^{ikr}}{4\pi r}\left(\hat{\theta}\hat{\theta}+\hat{\phi}\hat{\phi}\right)\cdot\int\mathbf{J}\left(\mathbf{r}'\right)e^{-ik\hat{\mathbf{r}}\cdot\mathbf{r}'}d\mathbf{r}' - ik\frac{e^{ikr}}{4\pi r}\hat{\mathbf{r}}\times\int\mathbf{M}\left(\mathbf{r}'\right)e^{-ik\hat{\mathbf{r}}\cdot\mathbf{r}'}d\mathbf{r}' \qquad (3)\\ &= ik\eta\frac{e^{ikr}}{4\pi r}\left[\left(\hat{\theta}\hat{\theta}+\hat{\phi}\hat{\phi}\right)\cdot\int\mathbf{J}\left(\mathbf{r}'\right)e^{-ik\hat{\mathbf{r}}\cdot\mathbf{r}'}d\mathbf{r}' - \hat{\mathbf{r}}\times\int\widetilde{\mathbf{M}}\left(\mathbf{r}'\right)e^{-ik\hat{\mathbf{r}}\cdot\mathbf{r}'}d\mathbf{r}'\right],\ k\mathbf{r}\gg 1\end{aligned}$$

where $\widetilde{\mathbf{M}} = \mathbf{M}/\eta$ and η is the wave impedance of free space.

If the source dimensions are small compared to a wavelength, we could use Taylor series to expand the phasor term $e^{-ik\hat{\mathbf{r}}\cdot\mathbf{r}'}$ in (3)

$$e^{-ik\hat{\mathbf{r}}\cdot\mathbf{r}'} = \sum_n \frac{(-ik)^n}{n!}(\hat{\mathbf{r}}\cdot\mathbf{r}')^n \approx 1 + (-ik)(\hat{\mathbf{r}}\cdot\mathbf{r}') \tag{4}$$

where the first-order Taylor expansion is employed. Consequently, the current integral is approximated as

$$\int\mathbf{J}\left(\mathbf{r}'\right)e^{-ik\hat{\mathbf{r}}\cdot\mathbf{r}'}d\mathbf{r}' \approx \int\mathbf{J}\left(\mathbf{r}'\right)d\mathbf{r}' + (-ik)\int\mathbf{J}\left(\mathbf{r}'\right)(\hat{\mathbf{r}}\cdot\mathbf{r}')d\mathbf{r}' \tag{5}$$

The first term of (5) is related to the electric dipole radiation

$$\int\mathbf{J}\left(\mathbf{r}'\right)d\mathbf{r}' = -\int\mathbf{r}'\left(\nabla'\cdot\mathbf{J}\left(\mathbf{r}'\right)\right)d\mathbf{r}' = -i\omega\int\mathbf{r}'\rho(\mathbf{r}')d\mathbf{r}' = -i\omega\mathbf{p}^e \tag{6}$$

where $\mathbf{p}^e = \int \mathbf{r}'\rho(\mathbf{r}')d\mathbf{r}'$ is the electric dipole moment. The second term of (5) correlates to the magnetic dipole and electric quadrupole radiations.

First, we can split the integrand into a symmetric and antisymmetric parts

$$(\hat{\mathbf{r}} \cdot \mathbf{r}')\mathbf{J} = \frac{1}{2}\left[(\hat{\mathbf{r}} \cdot \mathbf{r}')\mathbf{J} + (\hat{\mathbf{r}} \cdot \mathbf{J})\mathbf{r}'\right] + \frac{1}{2}(\mathbf{r}' \times \mathbf{J}) \times \hat{\mathbf{r}} \tag{7}$$

Then, we find the antisymmetric part can be expressed by magnetic dipole moment

$$\int \frac{1}{2}(\mathbf{r}' \times \mathbf{J}) \times \hat{\mathbf{r}} d\mathbf{r}' = -\hat{\mathbf{r}} \times \mathbf{m} \tag{8}$$

where $\mathbf{m} = \frac{1}{2}\int \mathbf{r}' \times \mathbf{J}\left(\mathbf{r}'\right) d\mathbf{r}'$ is the magnetic dipole moment. Next, the symmetric part can be expressed by electric quadrupole moment tensor

$$\begin{aligned} &\frac{1}{2}\int \left[(\hat{\mathbf{r}} \cdot \mathbf{r}')\mathbf{J}\left(\mathbf{r}'\right) + (\hat{\mathbf{r}} \cdot \mathbf{J}\left(\mathbf{r}'\right))\mathbf{r}'\right] d\mathbf{r}' \\ &= \frac{1}{2}\int \left(\mathbf{r}'\mathbf{J}\left(\mathbf{r}'\right) + \mathbf{J}\left(\mathbf{r}'\right)\mathbf{r}'\right) \cdot \hat{\mathbf{r}} d\mathbf{r}' \\ &= \frac{-i\omega}{2}\int \rho(\mathbf{r}')\mathbf{r}'\mathbf{r}' d\mathbf{r}' \cdot \hat{\mathbf{r}} = \frac{-i\omega}{6}\overline{\mathbf{Q}}^e \cdot \hat{\mathbf{r}} \end{aligned} \tag{9}$$

where $\overline{\mathbf{Q}}^e = \int (3\mathbf{r}'\mathbf{r}' - r'^2\overline{\mathbf{I}})\rho(\mathbf{r}')d\mathbf{r}'$ is the electric quadrupole moment tensor. Finally, the current integral can be rewritten as a summation of electric dipole, magnetic dipole, and electric quadrupole terms

$$\begin{aligned} &\int \mathbf{J}\left(\mathbf{r}'\right) e^{-ik\hat{\mathbf{r}}\cdot\mathbf{r}'} d\mathbf{r}' \approx \mathbf{J}_{ed} + \mathbf{J}_{md} + \mathbf{J}_{eq} \\ &= -i\omega\mathbf{p}^e + ik\hat{\mathbf{r}} \times \mathbf{m} - \frac{k\omega}{6}\overline{\mathbf{Q}}^e \cdot \hat{\mathbf{r}} \end{aligned} \tag{10}$$

Based on the dual principle, we have a similar expression for the magnetic current integral, i.e.

$$\begin{aligned} &\int \mathbf{M}\left(\mathbf{r}'\right) e^{-ik\hat{\mathbf{r}}\cdot\mathbf{r}'} d\mathbf{r}' \approx \mathbf{M}_{md} + \mathbf{M}_{ed} + \mathbf{M}_{mq} \\ &= -i\omega\mathbf{p}^m + ik\hat{\mathbf{r}} \times \mathbf{m}^e - \frac{k\omega}{6}\overline{\mathbf{Q}}^m \cdot \hat{\mathbf{r}} \end{aligned} \tag{11}$$

Supplementary Note 2: Power Conversion Efficiency of the Nonlinear Nanoantenna

The power conversion efficiency (η) of the nonlinear nanoantenna is defined as the output radiation power (P_o) at the second-harmonic frequency over the input power at the fundamental frequency (P_i).

$$\eta = \frac{P_o}{P_i} = \frac{\int_s \mathrm{Re}\left[\mathbf{E}^s(2\omega) \times \mathbf{H}^s(2\omega)\right] \cdot d\mathbf{S}}{\int_s \frac{|\mathbf{E}^i(\omega)|^2}{Z} dS} \tag{12}$$

where $Z = 120\pi$ is the wave impedance of free space. The effective focus area of laser is set to be 2 um $\times$ 2 um. **Figure S1** shows the power conversion efficiency of the nonlinear nanoantenna. From the figure, under a high input power ($P_i > 10^5$ W) condition, pump depletion effect should be taken into account.

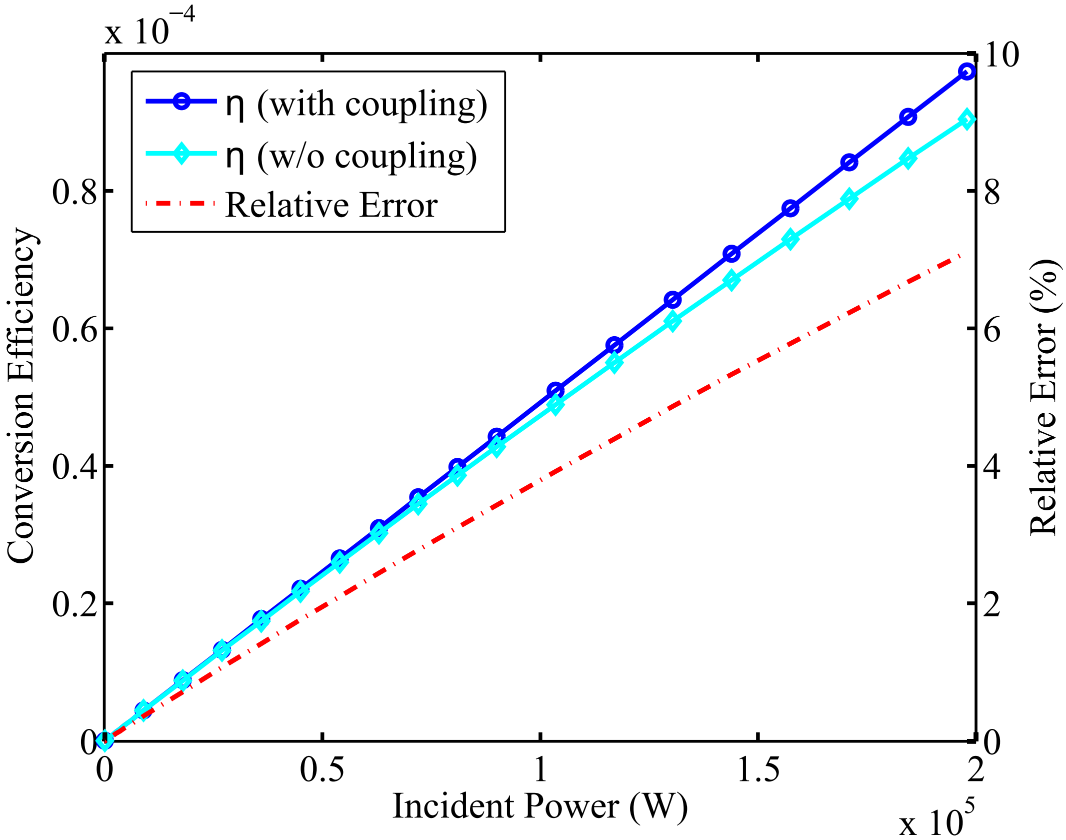

Figure 1: Power conversion efficiency of the nonlinear spherical nanoantenna. The undepleted pump approximation (w/o coupling case) is inaccurate under a high input power condition.