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The Application of Improved Periodic Wavelets Method for Scattering*

SHA Wei, WU Xianliang, CHEN Mingsheng and SUN Yufa

(*Key Laboratory of Intelligent Computing & Signal Processing, Anhui University, Hefei 230039, China*)

Abstract — Periodic Wavelets have found some applications to Method of moments (MoM) for Computational Electromagnetics. An improved method based on Fast Fourier transform (FFT) algorithm and Physical optics (PO) theory is presented in this paper. Through eliminating some superfluous periodic wavelet-basis functions, the dimensions and condition number of impedance matrix are reduced remarkably, thus makes fast and stable computation. Besides, speedy matrix filling and efficient matrix inversion also get desirable numerical results.

Key words — Periodic wavelets, Method of moments, Fast Fourier transform, Physical optics.

I. Introduction

With the properties of orthogonality, localization and multiresolution, wavelet transform has been applied to many fields like digital signal processing, data compression, communication coding, numerical calculation, and so on.

Wavelet transform used in Method of Moments is classified into discrete wavelet transform[1] and continuous wavelet transform[2]. The former directly makes impedance matrix sparse, and then uses iterative methods for the solution of matrix equation, the latter uses entire domain wavelet-basis functions to approximate unknown currents, and then converts integral equation into matrix equation through Galerkin or point-testing method.

Continuous wavelet transform faces three problems: edge treatment, redundancy, and efficient computation. Periodic wavelets[3], intervallic wavelets, and second-generation wavelets have overcome the first difficult problem. This paper aims at solving remnant two problems by discarding redundant wavelet-basis functions, fast filling impedance matrix and introducing iterative algorithm for matrix inversion.

This paper is organized as follows. Periodic wavelets theory is specified in Section II, followed by improved computation method about filling matrix and discarding redundant wavelets in Section III, numerical results are presented in Section IV. Finally, summary is outlined in Section V.

II. Periodic Wavelets Theory

1. Definition

A periodic MRA on [0, 1] can be constructed by periodizing the wavelet-basis functions as[3]

$$\varphi^p_{j,k} = \sum_{l\in z} \varphi_{j,k}(x+l), \quad j \geq 0, \quad 0 \leq k < 2^j \tag{1}$$

$$\psi^p_{j,k} = \sum_{l\in z} \psi_{j,k}(x+l), \quad j \geq 0, \quad 0 \leq k < 2^j \tag{2}$$

where superscript p stands for periodicity, $\varphi_{j,k}$ stands for scalets, and $\psi_{j,k}$ stands for wavelets.

The periodic orthonormal basis system in $L^2([0,1])$ is show in Eq.(3)

$$\begin{cases} f_0(x) = \varphi^p_{0,0}(x) = 1 \\ f_{2^m+n}(x) = \psi^p_{m,n}(x) \end{cases} \tag{3}$$

A function can be expanded into the terms of wavelets as

$$F(x) = \sum_{n=0}^{2^m-1} a_n f_n(x) \tag{4}$$

where a_n is the inner product of $F(x)$ and $f_n(x)$, m is the desired highest resolution level.

Daubechies periodic wavelets with 10-order varnishing moments can be seen in Fig.1.

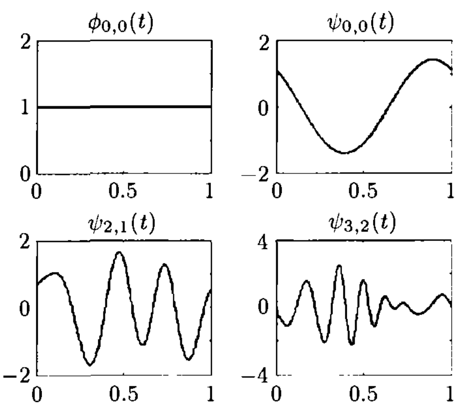

Fig. 1. Daubechies periodic wavelets with 10-order varnishing moments

2. Proper choice of wavelets

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Selecting right wavelet-basis functions[1] is a difficult problem for approximating unknown currents. Several elementary principles are summarized as follows.

In the first place, Continuous and smooth wavelets are suitable for smooth objects, while discontinuous wavelets are suitable for the objects including a large number of vertexes and edges. In the second place, the higher the order of vanishing moments is, the sparser the matrix or the smaller the dimensions become, but a longer computer time is required.

Commonly, 8-order or 9-order varnishing moments can accomplish desirable numerical results in the case of less computer time.

III. Improved Computation Method

1. General procedure

To demonstrate the improved solution method, we formulate the scattering problem of TM_Z electromagnetic plane by a perfectly conducting cylinder of a circular cross-section, using the E-field integral equation

$$\frac{\omega\mu_0}{4}\int_l J_z(r')H_0^{(2)}(k|r-r'|)dl' = E_z^{inc}(r), \quad r\in l \qquad (5)$$

where $E_z^{inc}(r)$ is the incident electrical field, J_z is the unknown induced current, $H_0^{(2)}$ is the second-type 0-order Hankel function, l is the perimeter of circular cross-section, ω is the angular frequency, μ_0 is the permeability of free space, and k is the wave number of free space. The cylinder is parallel to the z-axis, and the radius is a. Also, the incident wave is assumed to travel in the x direction.

Concrete procedure of solving the 2-D scattering problem by periodic wavelets is detailed as follows.

Step 1 Through polar coordinate and simple variable transform, Eq.(5) can be rewritten as

$$a\pi\frac{\omega\mu_0}{2}\int_0^1 J_z(\xi')H_0^{(2)}(2ka|\sin\pi(\xi-\xi')|)d\xi' = e^{-jka\cos(2\pi\xi)} \qquad (6)$$

Step 2 Expanding the unknown current in terms of periodic wavelets,

$$J_z(\xi') = \sum_{n=0}^{N} a_n f_n(\xi') \qquad (7)$$

and using point-testing procedure, we get the matrix equation

$$Z_{mn}I_n = V_m \quad . \qquad (8)$$

where

$$Z_{mn} = a\pi\frac{\omega\mu_0}{2}\int_0^1 f_n(\xi')H_0^{(2)}(2ka|\sin\pi(\xi_m-\xi')|)d\xi'$$

$$I_n = (a_1, a_2, \cdots a_n)^T, \quad V_m = e^{-jka\cos(2\pi\xi_m)}$$

Moreover, common analysis method or numerical method[4] can avoid the singularity of $\xi_m = \xi'$.

Step 3 Considering the number of testing points is ordinarily larger than that of wavelet-basis functions ($m > n$), an effective Recursive least square (RLS) algorithm can be found in Ref.[5].

2. Matrix filling

A fast matrix filling method ground on FFT algorithm is presented here. Z_{mn} in Eq.(8) can be described as

$$Z(:,n) = a\pi\Delta\xi\frac{\omega\mu_0}{2}IFFT(FFT(f_n(\xi)) \bullet FFT(H_0^{(2)}(2ka|\sin\pi\xi|))) \qquad (9)$$

where $Z(:,n)$ denotes matrix column vectors, $f_n(\xi)$ denotes periodic wavelet vectors, $H_0^{(2)}(2ka|\sin\pi\xi|)$ denotes Hankel vector, $FFT()$ denotes fast Fourier transform, $IFFT()$ denotes inverse fast Fourier transform, and $\bullet$ denotes inner product.

Assuming M wavelets and N discrete points are needed, fast Fourier transform only needs a final computational complexity of $O(MN\log_2(N))$ compared to that of $O(MN^2)$ for common integral algorithm.

3. Preconditioning method

As frequency increases, higher discretization and more wavelets will be necessary. But many wavelets are superfluous or redundant. Selecting these redundant periodic wavelets, results in ill-conditioned impedance matrix with enormous dimensions, excessive computer time and even bad accuracy. A new preconditioning method is presented here.

Firstly, from Physical optics (PO) theory, induced electric current in the surface of cylinder at high frequencies can be approximated as

$$\boldsymbol{J}_z = 2\boldsymbol{n}\times\boldsymbol{H}_y = \frac{2\boldsymbol{e}_r\times\boldsymbol{e}_y}{\boldsymbol{\eta}}\boldsymbol{E} = 2\frac{\boldsymbol{E}}{\boldsymbol{\eta}}\cos\phi\boldsymbol{e}_z, \quad \text{Illuminated area}$$

$$\boldsymbol{J}_z = 0, \quad \text{Shadow area} \qquad (10)$$

where $\boldsymbol{E}$ is electrical field, $\boldsymbol{\eta}$ is wave impedance in free space, and $\cos\phi$ is the angle vectors.

Secondly, the similarity between wavelets and electric current can be estimated by Eq.(11)

$$Corr = |<J_z, f_n(x)>| \qquad (11)$$

where $Corr$ denotes correlation coefficients, J_z denotes estimated unknown electric current vector, $f_n(x)$ denotes periodic wavelets vectors, $<,>$ denotes inner product, and $|\ |$ denotes modulus. Because of the circular shift property which periodic wavelets have, FFT algorithm also can be used in Eq.(11).

Thirdly, redundant wavelets with small correlation coefficients will be discarded by Eq.(12).

$$Corr < \gamma|MAX| \qquad (12)$$

where MAX is the maximum of correlation coefficients, γ is the detection threshold. Generally, γ is about 0.01.

IV. Numerical Results

To show the above analysis and efficiency of algorithm, some figures and tables are presented.

In Fig.2, induced current distribution and bistatic Radar cross section (RCS) are clearly given, where the perimeter of cylinder is one wavelength.

Vanishing moments' influence on the precision of bistatic RCS and condition number of impedance matrix is listed in Table 1. The relative error is computed according to

$$error = \frac{||RCS\_PeriodicWavelets - RCS\_MoM||_2}{||RCS\_MoM||_2} \qquad (13)$$

where $RCS\_PeriodicWavelets$ is the bistatic RCS computed by periodic wavelets, together with RCS_MoM computed by traditional MoM, and $\| \quad \|_2$ is 2-norm.

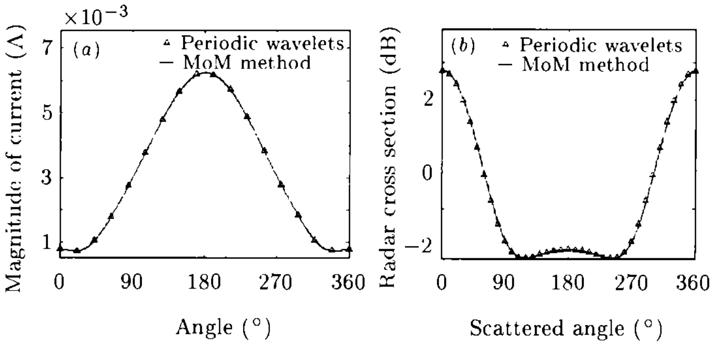

Fig. 2. Induced current distribution and bistatic RCS of cylinder

According to Table 1, the higher order the vanishing moments is, the lower error and condition number we get. However, when the radius equals to four wavelengths, condition number is not fit for other cases, because Green function's acute oscillation, the drastic change of induced current's phase and truncation error cause fictitious noise and redundant wavelets.

Table 1. Vanishing moments' influence on condition number of impedance matrix and the precision of bistatic RCS

Radius		$a=\lambda/4$	$a=\lambda$	$a=4\lambda$
Number of discrete points		256	256	512
Number of periodic wavelets		16	64	64
Condition number	2-order	16.48	36.59	12.22
	10-order	9.85	19.95	13.33
RCS numerical precision	2-order	0.45%	0.52%	1.49%
	10-order	0.34%	0.28%	0.89%

Table 2. Computer time comparisons on impedance matrix filling between FFT algorithm and general integral algorithm

Radius	$a=\lambda/4$	$a=\lambda$	$a=4\lambda$
Integral algorithm(s)	70.44	251.70	763.02
FFT algorithm(s)	0.33	1.26	1.92

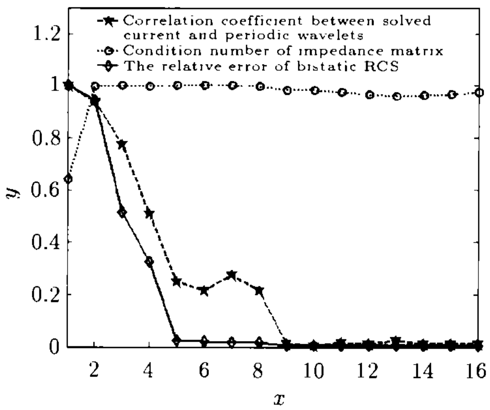

Fig. 3. The relationship between correlation coefficients, condition number of impedance matrix and relative error of bistatic RCS

Different computer time comparisons between FFT algorithm and general quadrature method are illustrated in Table 2.

Some curves about preconditioning method at low frequency are shown in Fig.3. We discard every wavelet in turn, sequentially recompute condition number and relative error by other remnant wavelets. Correlation coefficients estimated by MoM are also plotted for comparison. In Fig.3, the perimeter is one wavelength, the number of periodic wavelets is 16, the first wavelet is $\varphi_{0,0}^{p}$, and all curves have been normalized to the maximum value of unity.

Numerical results at high frequency can be found in Table 3, where the radius becomes ten wavelengths, and 1024 discrete points are used.

Table 3. Numerical results about preconditioning periodic wavelets method, traditional periodic wavelets method, and physical optics method

Different method comparison	PO	Traditional periodic wavelets	Preconditioning periodic wavelets
Number of periodic wavelets	/	256	159
Time for inversion (second)	/	11.33	4.75
Condition number of matrix	/	172.13	14.23
Bistaic RCS relative error	9.69%	0.94%	0.99%

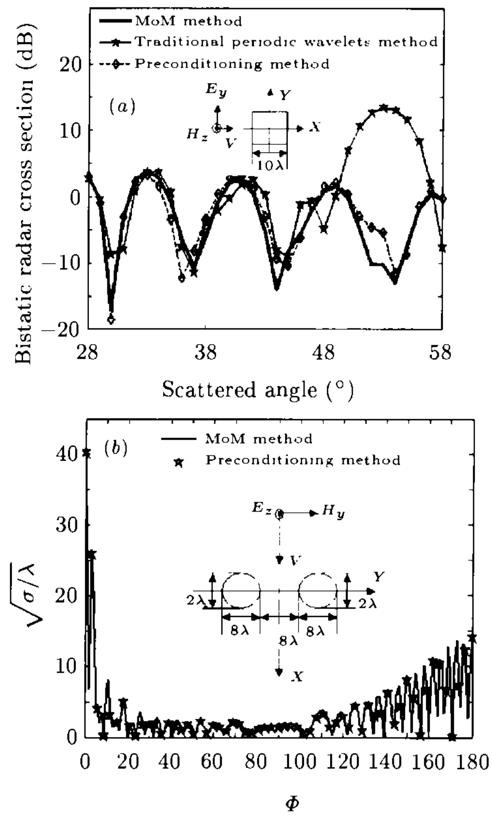

Fig. 4. (a) Bistatic RCS comparisons about TE scattering; (b) Bistatic RCS comparisons about TM multiple scattering

Other cases can be found in Fig.4. In Fig.4(a), the scattering of TE_Z plane wave from conducting square-column is computed through MFIE. Partial scattered angles are plotted for comparison. 50 wavelets are discarded and better accuracy is accomplished contrasted to traditional periodic wavelets method. The illustration of multiple scattering by two conducting elliptic cylinder is shown in Fig.4(b). Smaller wavelets are used through preconditioning method and half condition number is reduced.

V. Summary

The application of periodic wavelets for MoM method can resolve the 2-D scattering or radiation problem, when symmetrical currents distribution in the surface or volume of scattered object is realized.

Improved methods based on FFT algorithm and PO theory accomplish better efficiency or accuracy, and may be applied to traditional wavelets, intervallic wavelets or lazy wavelets for electromagnetic computation.

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SHA Wei is a graduate student in electromagnetic field and microwave technique specialty of Anhui University. His research interests are in the areas of electromagnetic scattering, numerical technique in electromagnetics, and wavelet theory.

WU Xianliang professor, Ph.D. supervisor. Graduated from Electronic Department of Anhui University in 1982. He has engaged in teaching and research on the theory of electromagnetic field and microwave in Electronic Department of Anhui University since 1984. He is a senior member of the Chinese Institute of Electronics. He has complied and published ≪Antenna Theory and Engineering≫ and ≪Electromagnetic Scattering Theory and Computation≫ etc. He has written and published more than 60 papers.

CHEN Mingsheng is a graduate student in electromagnetic field and microwave technique specialty of Anhui University. His research interests are in the areas of electromagnetic scattering, numerical technique in electromagnetics, and digital signal processing.